Gebrochen rationale Funktionen

Mathematik in wxMaxima www.mathematik-verstehen.de Haftendorn Nov 2010

0.1 Handlinghilfen

0.2 Definition
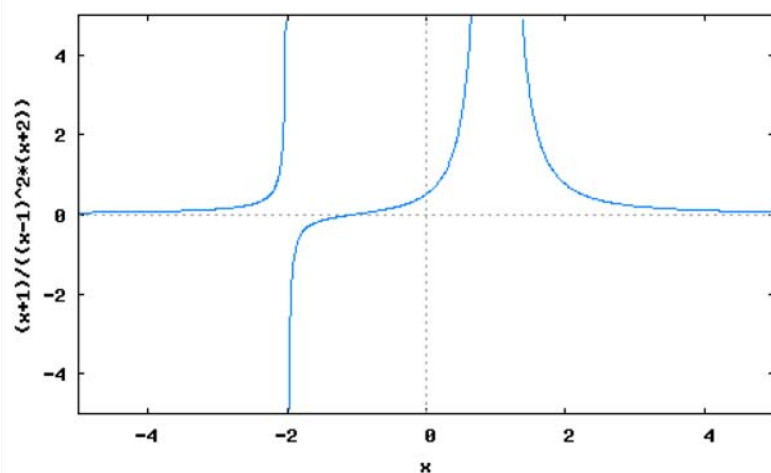
(%i6) h(x):=(x+1)/((x-1)^2*(x+2));

(%o6) $h(x):=\dfrac{x+1}{(x-1)^2\,(x+2)}$

--> wxplot2d([h(x)], [x,-5,5],[y,-5,5])$

plot2d: some values were clipped.

(%t2)

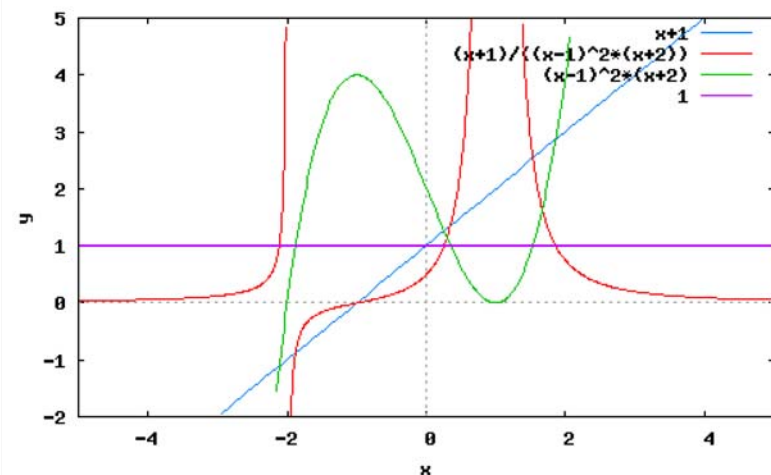(%i7) num(h(x));denom(h(x));

(%o7) $x+1$

(%o8) $(x-1)^2\,(x+2)$

(%i14) wxplot2d([num(h(x)),h(x),denom(h(x)),1], [x,-5,5], [y,-2,5])$

plot2d: some values were clipped.
plot2d: some values were clipped.
plot2d: some values were clipped.

(%t14)

1 Partialbruchzerlegung

1.1 Schnelle Lösung

--> hh(x):=partfrac((x+1)/((x-1)^2*(x+2)),x);hh(x);

(%o3) $hh(x):=\text{partfrac}\left(\dfrac{x+1}{(x-1)^2\,(x+2)},x\right)$

(%o4) $-\dfrac{1}{9\,(x+2)}+\dfrac{1}{9\,(x-1)}+\dfrac{2}{3\,(x-1)^2}$

1.2 Elemente der von-Hand-Arbeit

--> glei:x+1=A*(x-1)*(x+2)+B*(x+2)+C*(x-1)^2;

(%o5) $x+1=(x-1)^2\,C+(x+2)\,B+(x-1)\,(x+2)\,A$

--> glei;

(%o6) $x+1=(x-1)^2\,C+(x+2)\,B+(x-1)\,(x+2)\,A$

--> eins:subst(1,x,glei);
zwei:subst(-2,x,glei);
drei:subst(0,x,glei);

(%o7) $2=3\,B$

(%o8) $-1=9\,C$

(%o9) $1=C+2\,B-2\,A$

--> lo:solve([eins,zwei,drei],[A,B,C]);

(%o10) $[[A=\dfrac{1}{9},B=\dfrac{2}{3},C=-\dfrac{1}{9}]]$

--> lo[1];

(%o11) $[A=\dfrac{1}{9},B=\dfrac{2}{3},C=-\dfrac{1}{9}]$

Einsetzung

--> subst(lo[1],glei);

(%o12) $x+1=\dfrac{(x-1)\,(x+2)}{9}+\dfrac{2\,(x+2)}{3}-\dfrac{(x-1)^2}{9}$

Probe

--> rhs(%);factor(%);

(%o13) $\dfrac{(x-1)\,(x+2)}{9}+\dfrac{2\,(x+2)}{3}-\dfrac{(x-1)^2}{9}$

(%o14) $x+1$

1.3 Koeffizientenvergleich

--> `x+1=expand(A*(x-1)*(x+2)+B*(x+2)+C*(x-1)^2);`

(%o15) $x + 1 = x^2 C - 2 x C + C + x B + 2 B + x^2 A + x A - 2 A$

Mit Hinsehen eintragen

--> `solve([0=C+A,1=-2*C+B+A,1=C+2*B-2*A],[A,B,C]);`

(%o16) $[[A = \frac{1}{9}, B = \frac{2}{3}, C = -\frac{1}{9}]]$

Dasselbe Ergebnis wie oben.

1.4 Zählergrad > Nennergrad

--> `k(x):=(x+4)*(x-3)*(x-7)*(x+1)/((x-1)^2*(x+2));`

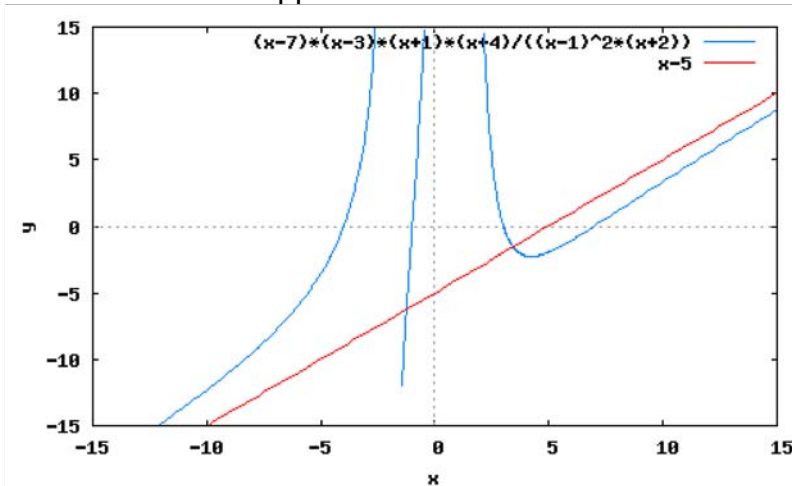(%o17) $k(x) := \frac{(x + 4)(x - 3)(x - 7)(x + 1)}{(x - 1)^2 (x + 2)}$

--> `partfrac(k(x),x);`

(%o18) $-\frac{10}{x + 2} + x - \frac{12}{x - 1} + \frac{40}{(x - 1)^2} - 5$

--> `wxplot2d([k(x),x-5], [x,-15,15], [y,-15,15])$`

plot2d: some values were clipped.
plot2d: some values were clipped.

(%t19)

--> `integrate(k(x), x);`

(%o20) $-10 \log(x + 2) - 12 \log(x - 1) + \frac{x^2 - 10 x}{2} - \frac{40}{x - 1}$

1.5 Weiteres Beispiel (vom Übersichtsblatt)

(%i2) `f1s(x):=(x-1)^2*(x+1)/((x+2));`
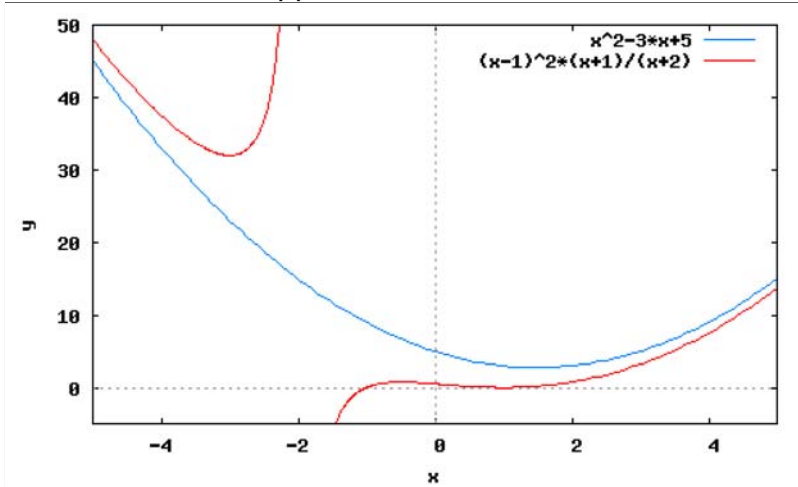
(%o2) $f1s(x) := \frac{(x - 1)^2 (x + 1)}{x + 2}$

--> partfrac(f1s(x),x);

(%o22) $-\dfrac{9}{x+2} + x^2 - 3\,x + 5$

--> wxplot2d([x^2-3*x+5,f1s(x)], [x,-5,5], [y,-5,50])$

plot2d: some values were clipped.

(%t23)
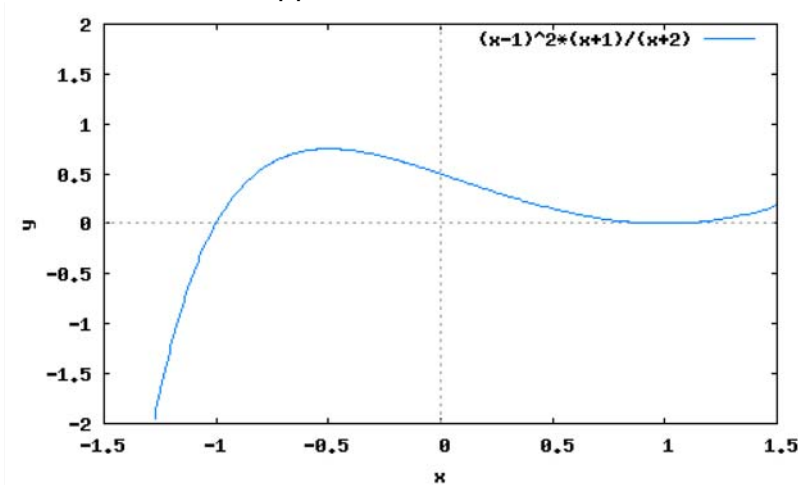

--> wxplot2d([x^2-3*x+5,f1s(x)], [x,-1.5,1.5],[y,-2,2])$

plot2d: all values were clipped.
plot2d: some values were clipped.

(%t24)

--> integrate(f1s(x), x, -1, 1);float(%);

(%o25) $\dfrac{41}{6} - \dfrac{54\,\log(3) - 23}{6}$

(%o26) 0.77915606865368

(%i4) num(f1s(x));denom(f1s(x));

(%o4) $(x-1)^2\,(x+1)$

(%o5) $x+2$