logarithmische Spirale, durch Kreisbögen angenähert

siehe dazu eine Geogebra-Datei: logarithmischeSpirale-mitKreisen.ggb (Autor: Prof. Dr. Dieter Riebesehl, Leuphana Universität Lüneburg)

- **der goldene Schnitt**

$$\phi = \frac{1+\sqrt{5}}{2};$$

- **Punkt F und Punkt G (Zentrum der Spirale), Parameter der Spirale**

$$\text{solG} = \text{Solve}\left[\left\{\frac{a}{b} == \phi - 1,\ \frac{b}{1-a} == \phi - 1\right\}\right][[1]] \text{ // Simplify}$$

$$\left\{a \to \frac{1}{10}\left(5-\sqrt{5}\right),\ b \to \frac{1}{\sqrt{5}}\right\}$$

$$\text{F} = \{b - \phi + 1,\ 1 - a\} \text{ /. solG // Simplify}$$

$$\left\{\frac{1}{10}\left(5-3\sqrt{5}\right),\ \frac{1}{10}\left(5+\sqrt{5}\right)\right\}$$

```
F // N
```

{−0.17082, 0.723607}

$$x[\alpha\_] := s\,\phi^{\frac{\alpha}{\pi/2}}\,\text{Cos}[\alpha]$$

$$y[\alpha\_] := s\,\phi^{\frac{\alpha}{\pi/2}}\,\text{Sin}[\alpha]$$

```
parms = FindRoot[{x[α] == F[[1]], y[α] == F[[2]]}, {{α, π / 2}, {s, 1}}]
```

{α → 1.80262, s → 0.428004}

$$\alpha \Big/ \left(1\,°\right) \text{ /. parms}$$

103.283

- **Rumspielen mit algebraischen Zahlen: wo genau ist die Spiralmitte G**

```
woa = ToNumberField[a /. solG, ϕ]
```

$$\text{AlgebraicNumber}\left[\frac{1}{2}\left(1+\sqrt{5}\right),\ \left\{\frac{3}{5},\ -\frac{1}{5}\right\}\right]$$

```
wob = ToNumberField[b /. solG, ϕ]
```

$$\text{AlgebraicNumber}\left[\frac{1}{2}\left(1+\sqrt{5}\right),\ \left\{-\frac{1}{5},\ \frac{2}{5}\right\}\right]$$

```
woa[[2]].{1, ϕ // HoldForm}
```

$$\frac{3}{5} - \frac{\phi}{5}$$

```
wob[[2]].{1, ϕ // HoldForm}
```

$$-\frac{1}{5}+\frac{2\,\phi}{5}$$

- **Ein weiterer Punkt auf der Spirale ist T, der Schnitt des Kreises im Quadrat ABEF mit der Diagonalen g im Rechteck ABCD**

```
B = {-ϕ + b, -a} /. solG // Simplify
```

$$\left\{\frac{1}{10}\left(-5-3\sqrt{5}\right),\ \frac{1}{10}\left(-5+\sqrt{5}\right)\right\}$$

```
lf = F.F // Simplify
```

$$1-\frac{1}{\sqrt{5}}$$

```
lb = B.B // Simplify
```

$$1+\frac{1}{\sqrt{5}}$$

```
Ep = {b - ϕ + 1, -a} /. solG // Simplify
```

$$\left\{\frac{1}{10}\left(5-3\sqrt{5}\right),\ \frac{1}{10}\left(-5+\sqrt{5}\right)\right\}$$

```
g[x_] := -(ϕ - 1) x
```

```
solT = Solve[Norm[{x, g[x]} - Ep] == 1, x][[1]] // Simplify
```

$$\left\{x\to-\sqrt{\frac{1}{5}+\frac{1}{\sqrt{5}}}\right\}$$

```
T = {x, g[x]} /. solT // Simplify
```

$$\left\{-\sqrt{\frac{1}{5}+\frac{1}{\sqrt{5}}},\ \frac{1}{2}\sqrt{\frac{1}{5}+\frac{1}{\sqrt{5}}}\left(-1+\sqrt{5}\right)\right\}$$

- $|T|^2 = |F|\ |B|$

```
lt = T.T // Simplify
```

$$\frac{2}{\sqrt{5}}$$

```
lt^2 - lb lf // Simplify
```

0

- **Die Gerade g halbiert den Winkel FGB**

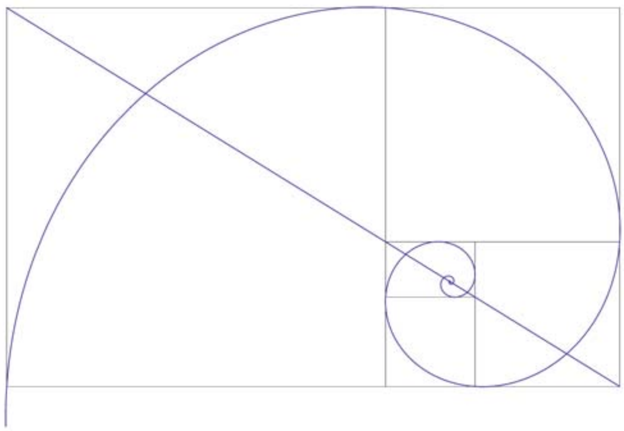
$$\frac{T.F}{\sqrt{T.T}\ \sqrt{F.F}}\ //\ \mathrm{Simplify}$$

$$\frac{1}{\sqrt{2}}$$

Die logarithmische Spirale

- **Vorbereitung**
- **Bild**

```
Show[quads, curve, geradeg]
```



- **Die Spirale passt nicht ganz in die Quadrate**

```
top = FindRoot[Y'[α] == 0, {α, π / 2}]
```

{α → 1.86807}

```
t1 = Y[α] /. top
```

0.725283

```
t1 / t2
```

$\frac{0.725283}{t2}$

```
t2 = X[α] /. top
```

-0.22219

```
Show[quads, curve,
 PlotRange → {{t2 - 0.1, F[[1]] + 0.03}, {F[[2]] - 0.03, t1 + 0.01}}, AspectRatio → Automatic]
```