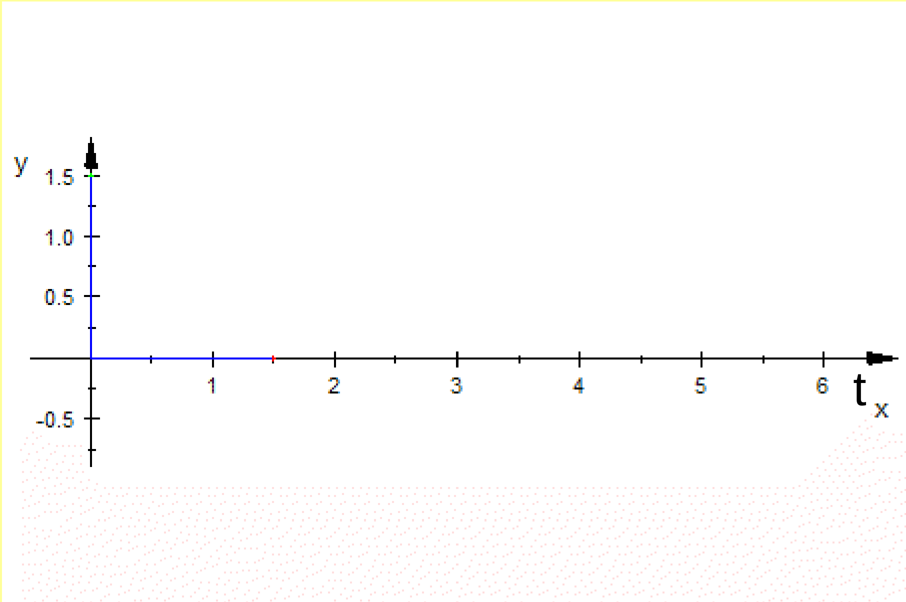
Polar Coordinates in Double-Perspective

by movable and simultaneous visualisation of the respective „cartesian function"

$$r = r(t) = \cos(t) + \frac{1}{2}$$



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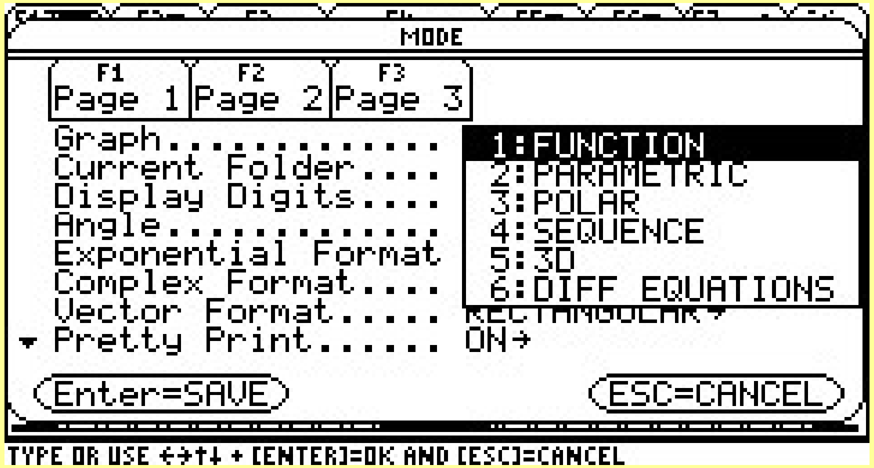
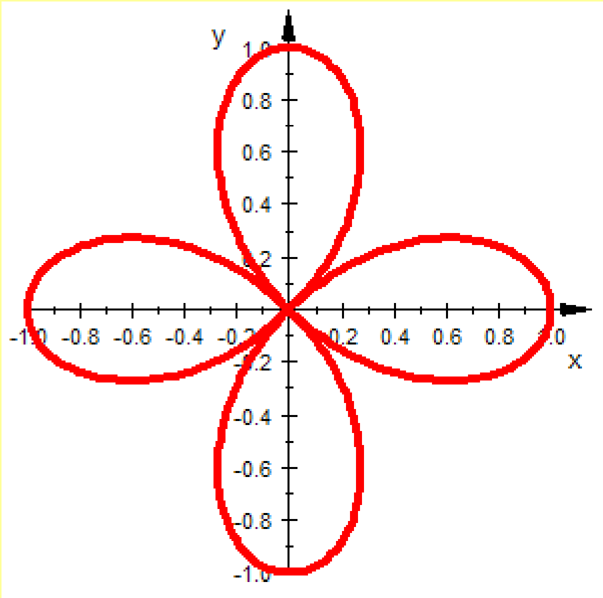
Why should we teach polar coordinates?

- they enable wonderful mathematics
- they open up a part of the world
- the students can explore on their own
- because Günter Steinberg gave 1000 reasons decades ago
- ....

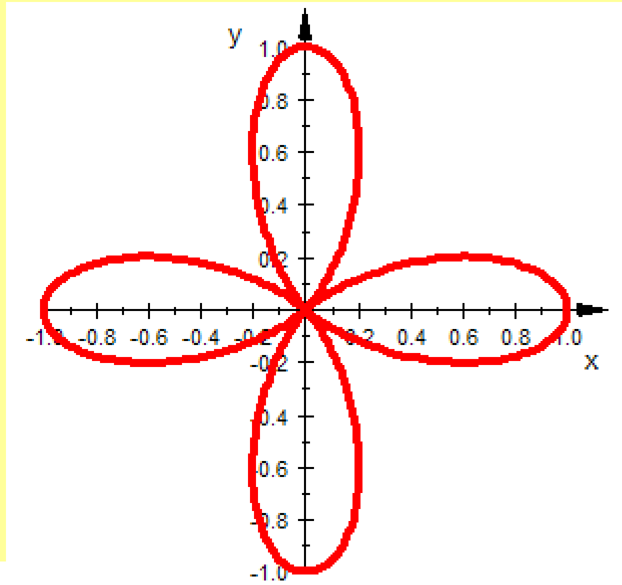


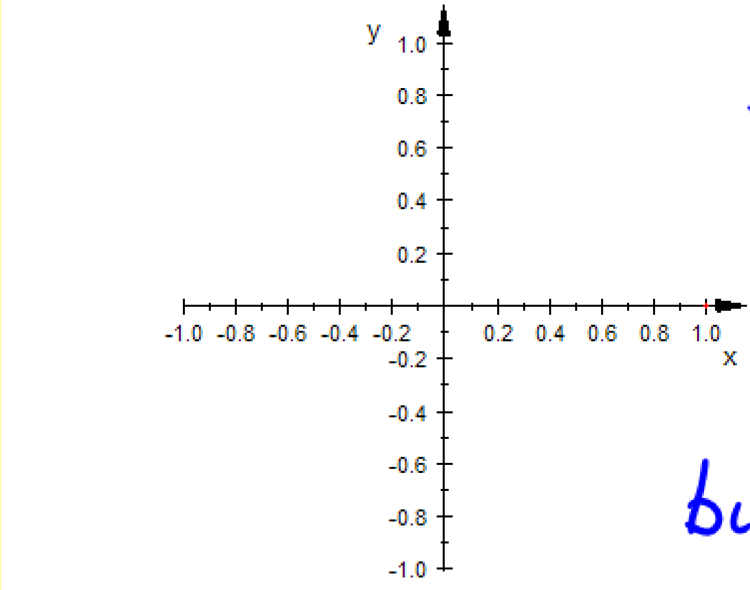
- We should provide an explanation for the meaning of these menu items

But there is more to it!



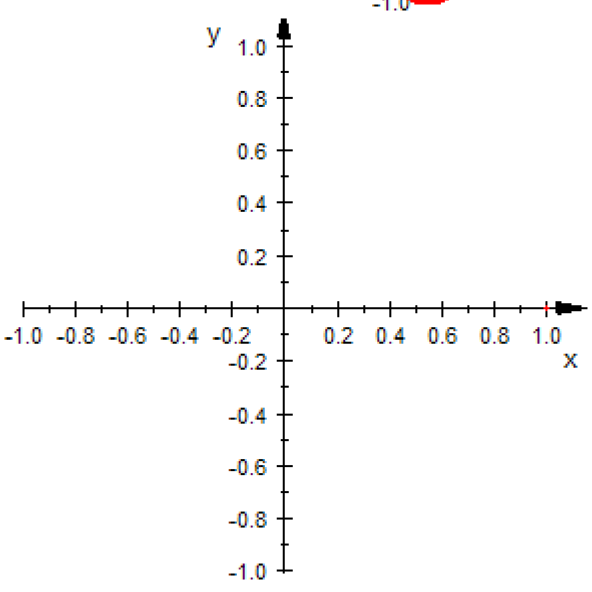
What is the sense of progression in the die cosine-rosettes?





see!

but why?



What can you expect from this lecture?

- introduction

the answer!

- the particular idea
  - presented with several tools
- examples of usage
- further developments
- the potentialities for the learning of mathematics
- conclusion

and you can find all that on the internet

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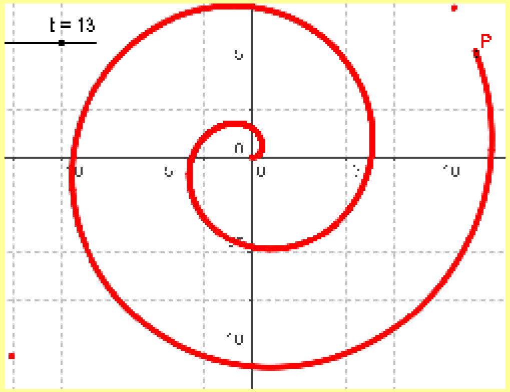
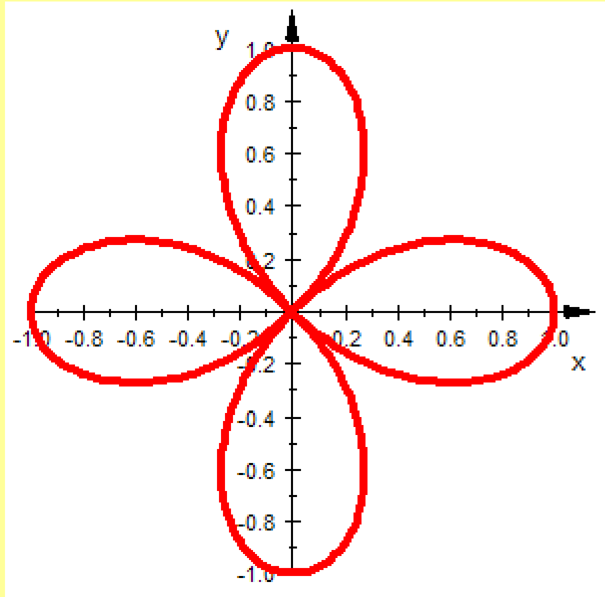
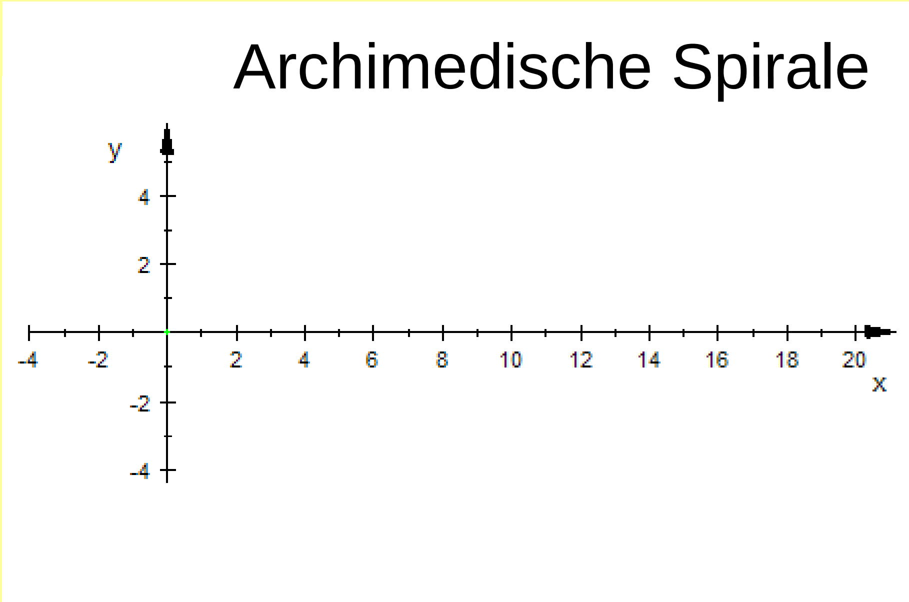
How to understand the sense of progression?







GeoGebra

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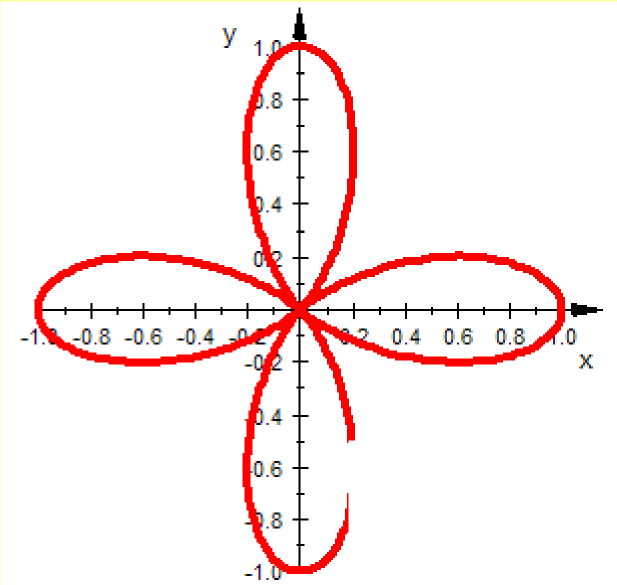
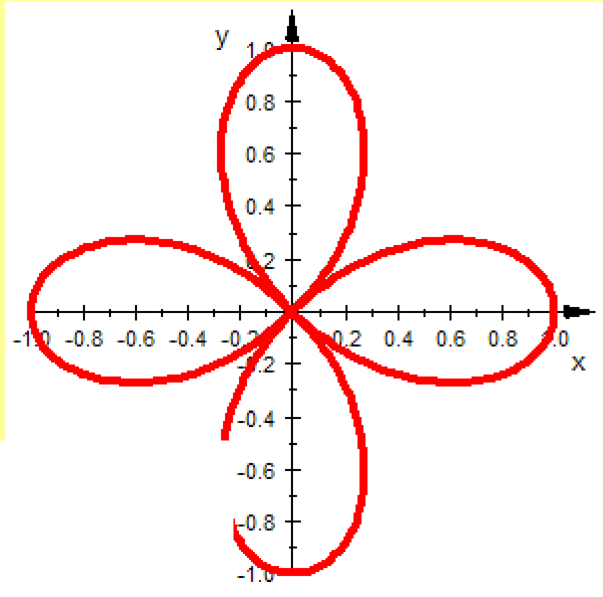
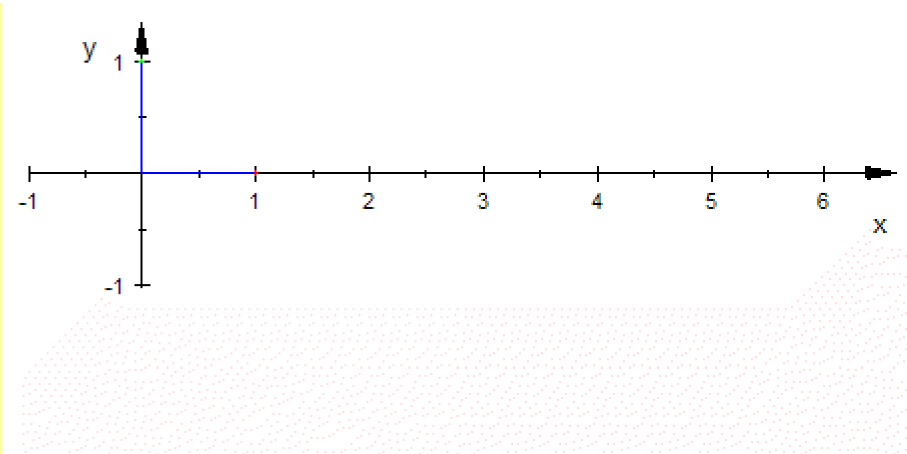
polar-cartesian-double-perspective







GeoGebra

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polar-cartesian-double-perspective is more difficult with Euklid-Dynageo

Spiral

Cos(2t)
Polar-kartsesisch
Noch eine Panne

Sin-Panne

Cos-Panne

Cos richtig

In Euklid-Dynageo bekommt man leicht Probleme mit dem „Punktsprung-Phänomen“

In Euklid-Dynageo erfordern die trigonometrischen Funktionen als Argument Winkel im Gradmaß.

Cos(2t) Polar-kartesesisch
Dieses stimmt.

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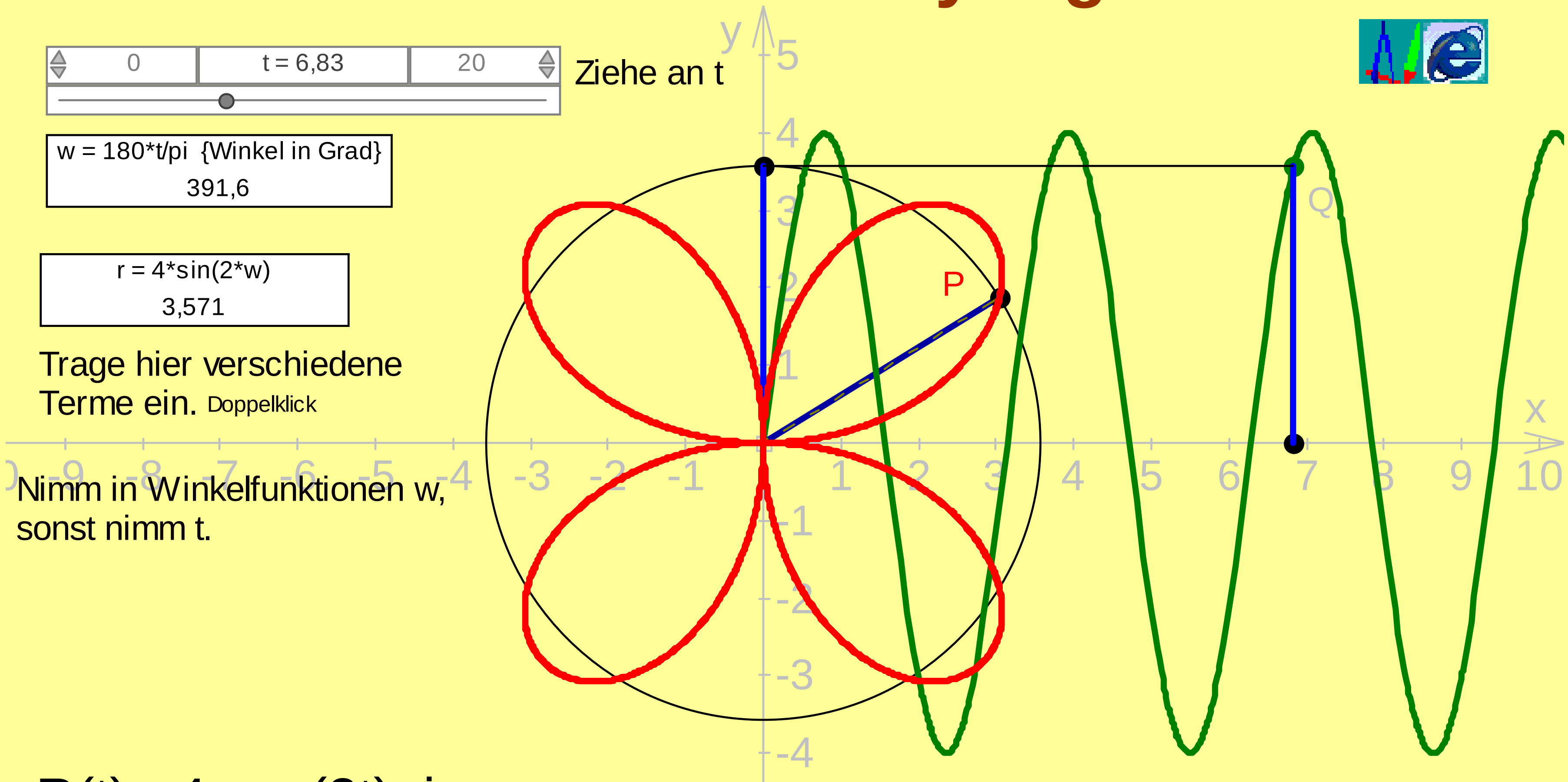
polar-cartesian-double-perspective with Euklid-Dynageo



R(t)= 4 cos(2t) in **polar-cartesisian-double-perspective**

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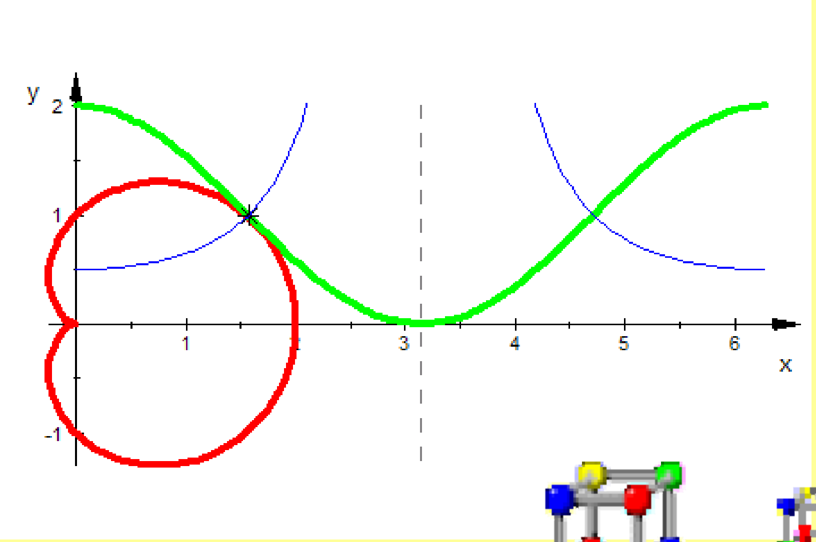
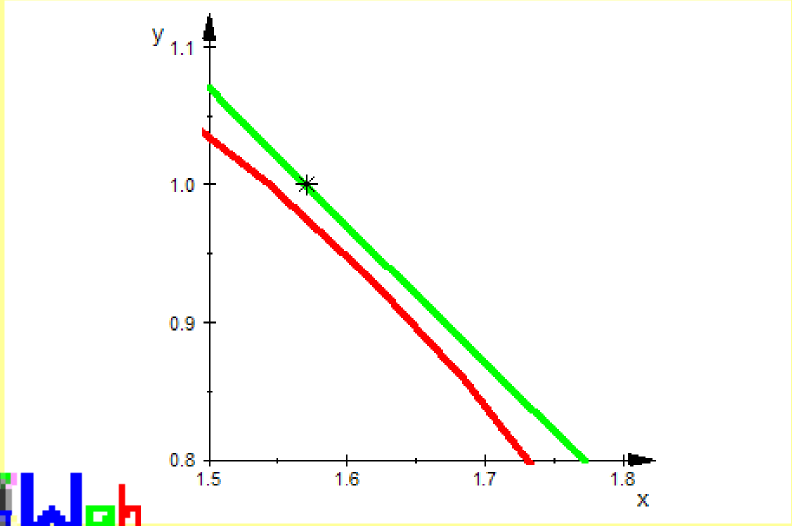
polar-cartesian-double-perspective with MuPAD

experimenting with MuPAD

related site





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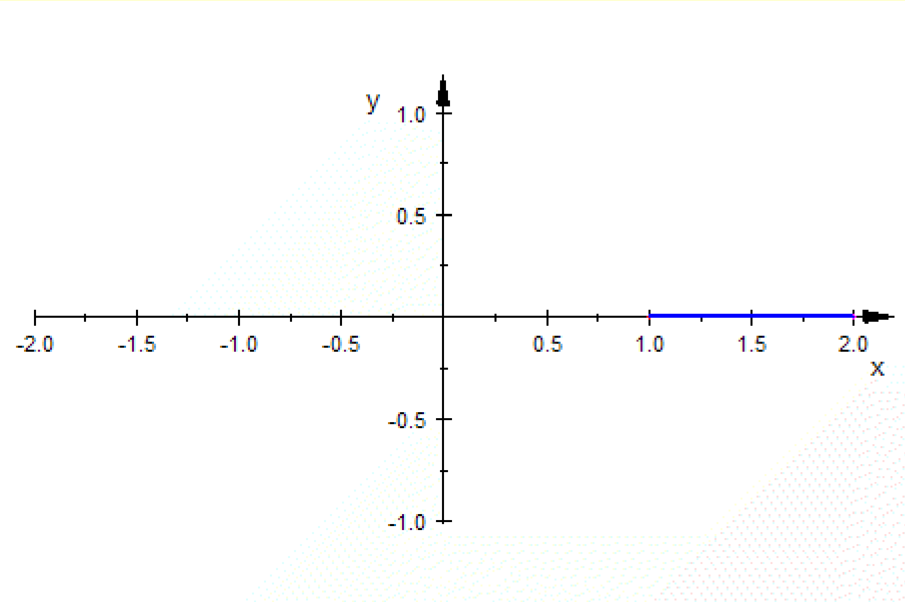
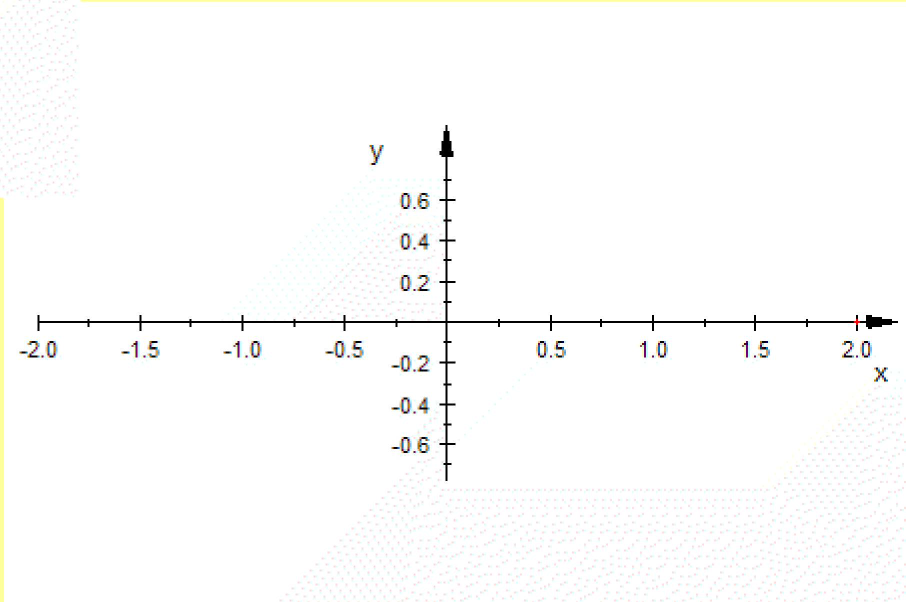
A conchoide of the cosine rosette...



$$r(t) = \cos(2t) + 1$$

..is the double-egg-line

$$r(t) = 2\cos(t)^2$$



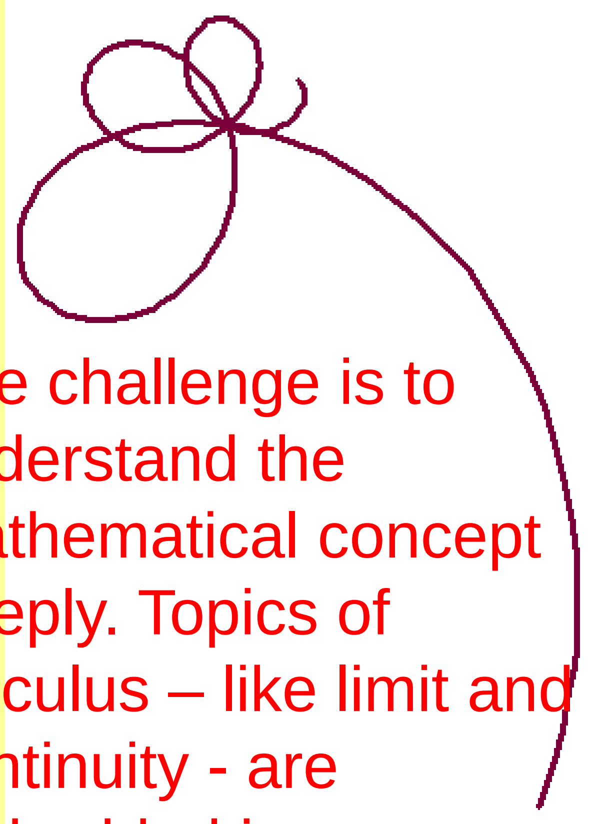
Polar flower (state examination problem)

Es ist $r(\varphi) = \frac{1}{\varphi - 1}\cos\left(\frac{\pi}{2}\varphi\right)$ gegeben.

a) Entwickeln Sie einen kartesischen Graphen hierzu aus zwei Bausteinen für $\varphi \geq 0$. (Siehe auch Teil c)

b) Rechts ist der Polar-Graph im Intervall [2, 10] dargestellt. Zeichnen Sie rechts ein Koordinatensystem ein und bestimmen Sie für Anfangs- und Endpunkt Gradmaß, Radius und kartesische Koordinaten.
Kennzeichnen Sie in Ihrem Bild aus a) die Entsprechungen dieser drei Blätter.

c) Bestimmen Sie exakt $\lim_{\varphi \to 1} r(\varphi)$.
Warum zeigt der TI im 3. Quadranten eine Lücke? Zeichnen Sie hier (durch den Text hindurch) die Polarblume von $\varphi = 0$ an.
Wie sieht sie für immer größer werdende Winkel aus?



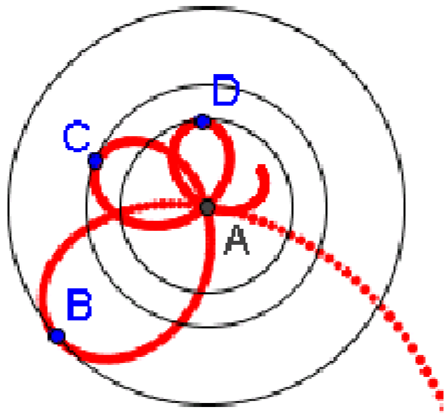
The challenge is to understand the mathematical concept deeply. Topics of calculus – like limit and continuity - are embedded in a non-standard context.

Polar flower (Staatsexam. Aufgabe)

d) **Bestimmen Sie numerisch mit TI und mit dem Keplerverfahren den Flächeninhalt des größten der oben dargestellten Blätter.**
Ermitteln Sie auch einen groben Näherungswert durch Einzeichnen und Auswerten einer elementaren Figur.
Erkären Sie, warum das Keplerverfahren hier keinen sonderlich guten Wert liefert.

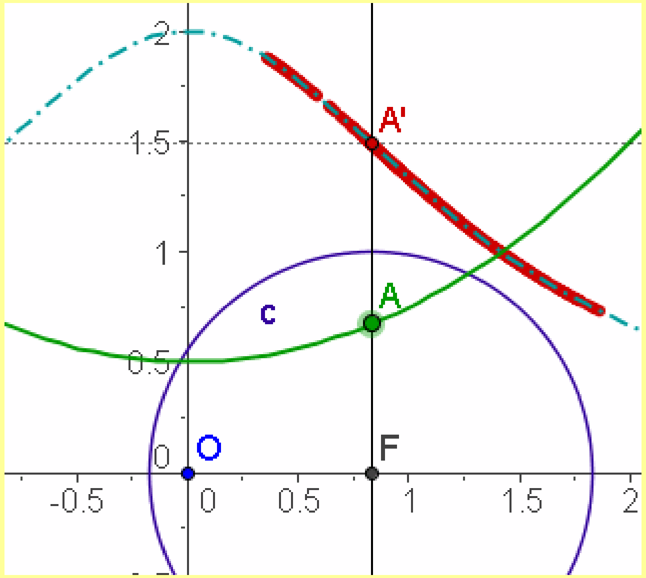
e) **Bestimmen Sie für dieses Blatt den vom Ursprung am weitesten entfernten Punkt als relatives Maximum von r (mit Ableitung von Hand, numerische Auswertung der Ableitung mit TI).**
Welche Möglichkeiten haben Sie, ohne Ableitung an numerische Werte zu kommen?
Warum kann man keine exakten Werte anstreben?

f) **In einem Dynamischen-Mathematik-System könnte man in der gezeigten Art Kreise „aufziehen". Begründen Sie, warum es für jedes Blatt genau einen solchen „Berührkeis" gibt.**
Beziehen Sie dies auf Ihre bisherige Aufgabenbehandlung.
Was lässt sich zu der Folge der Kreisradien und der Folge der Winkelstellungen der Berührpunkte B, C, D,... sagen?



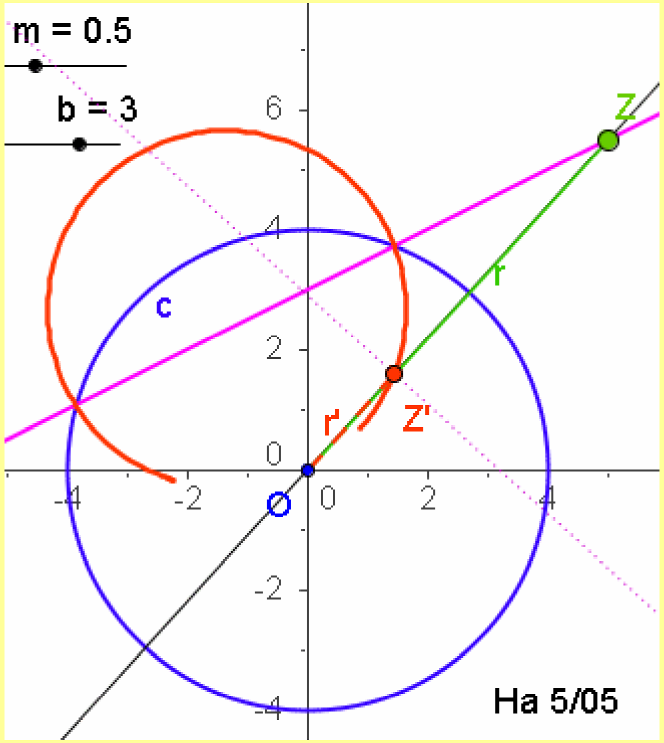
Computer tools for mathmatics are used in all my lessons.
Here the Students can show their competence.

Inversion at the unit circle



$$r = r(\varphi)$$

$$\tilde{r} = \frac{1}{r(\varphi)}$$



$$y = f(x)$$

$$y = \frac{1}{f(x)} \qquad z.B. \quad r = \cos(\varphi) \qquad \tilde{r} = \frac{1}{\cos(\varphi)}$$

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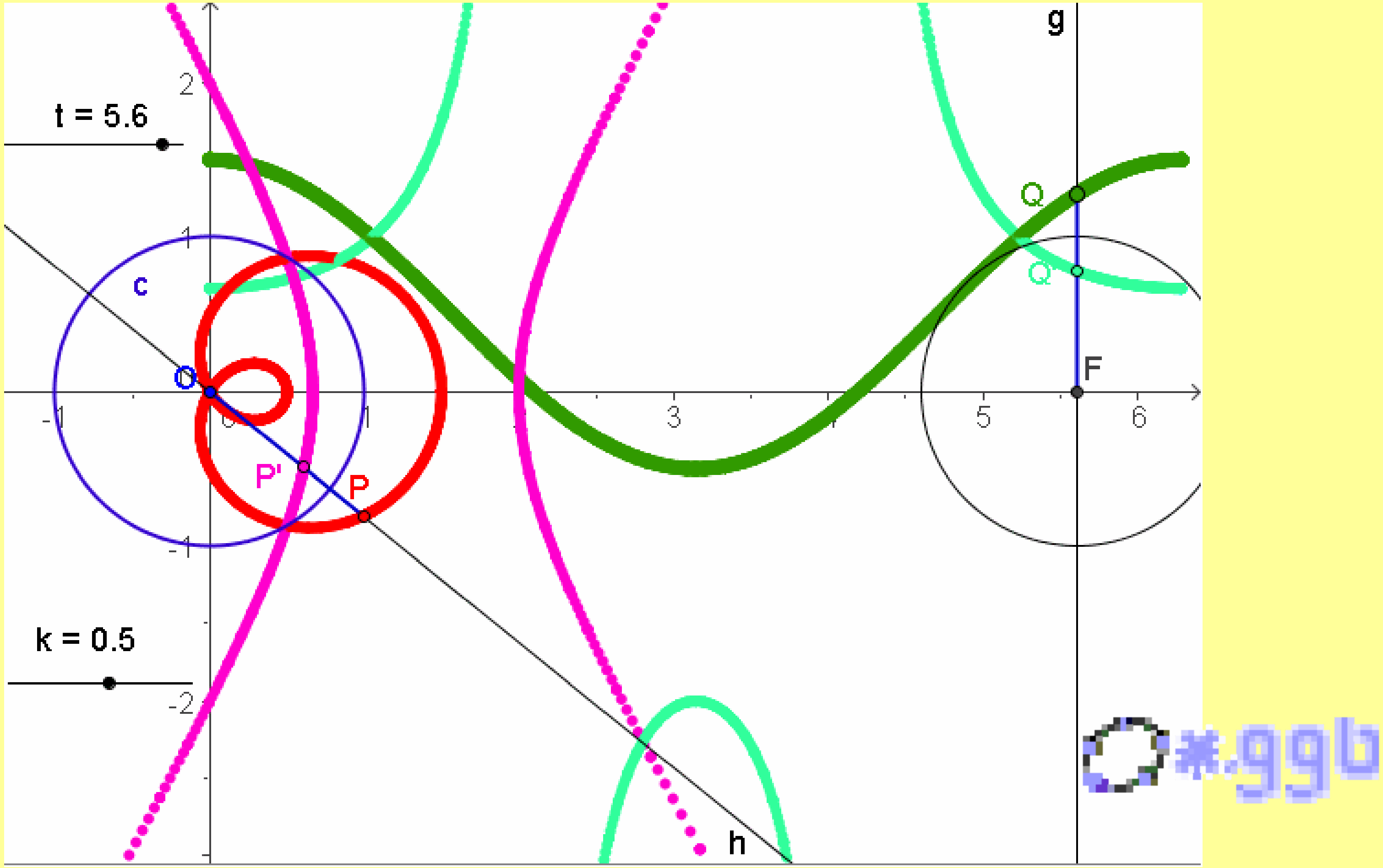
Inversion of the Pascal-Snails



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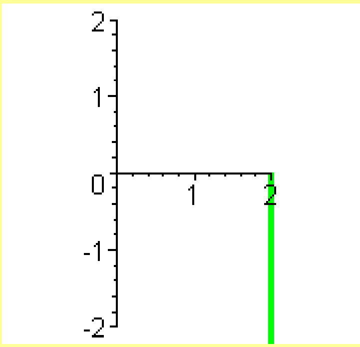
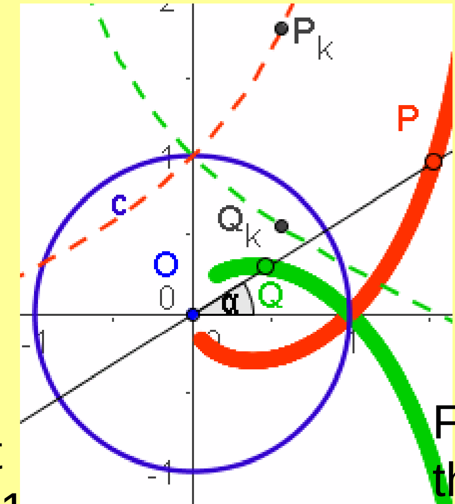
Inversion of the Strophoide

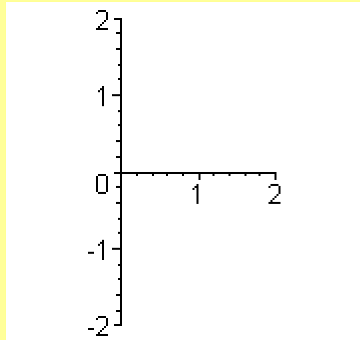
The green and the red curve are inverse of each other. The product of the terms is 1

$$r(t)=\frac{1-\sin(t)}{\cos(t)}$$

$$r(t)=\frac{1+\sin(t)}{\cos(t)}$$







For angles in the 1. quadrant
the radius is less or equal to 1

For angles in the 1. quadrant
the radius is greater or equal to 1

The strophoide is an **analagmatic curve:** it is fixed under inversion

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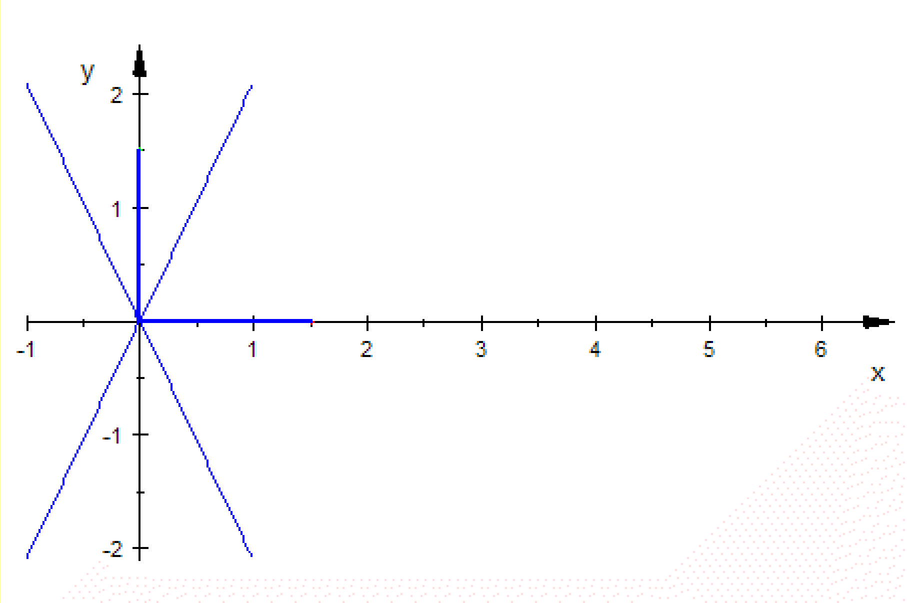
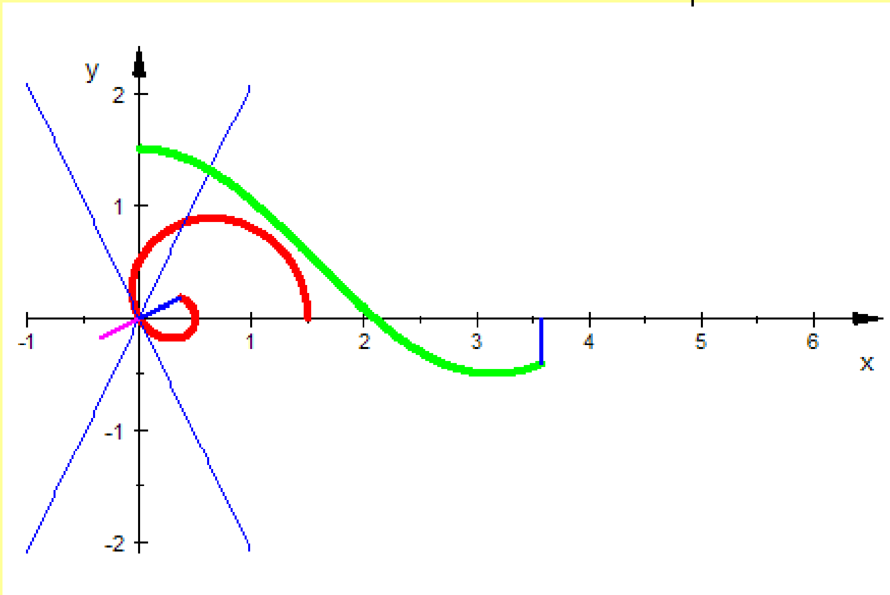
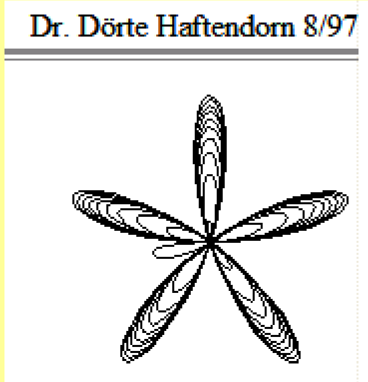
Analysis can be understood better





The roots in the cartesian perspective give
the slopes of the polar-curve at the origin.

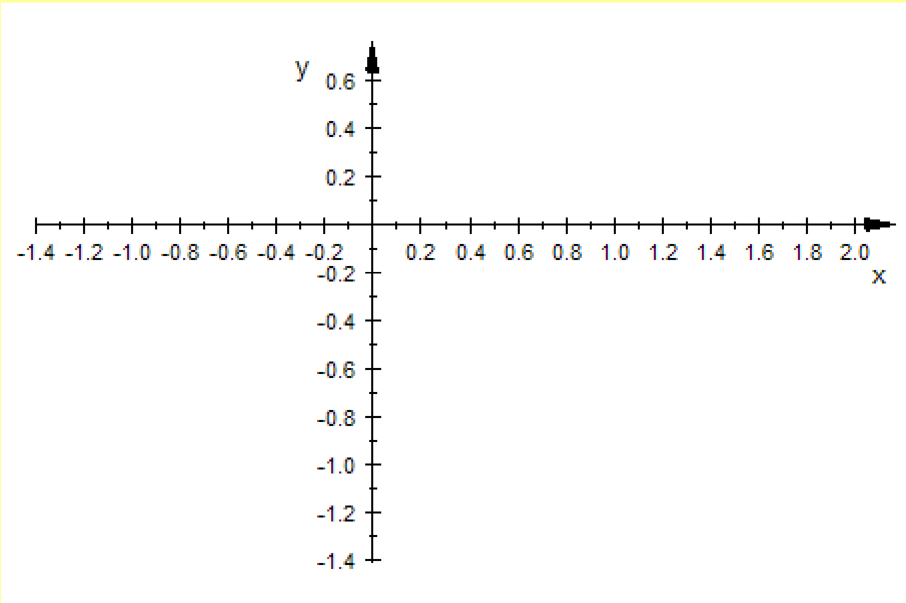
Stating problems and finding solutions



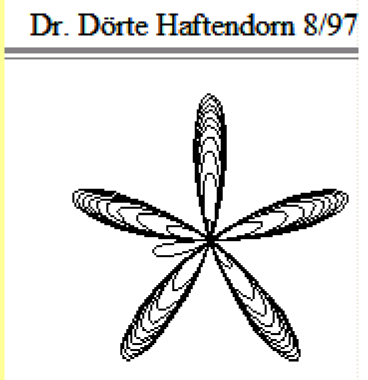
I published this problem in the book
„Analysis Aufgaben“ von Steinberg/ Ebenhöh

$$r(t) = \ln(t)\,\sin(5t)$$

Where does the little extra petal come from?



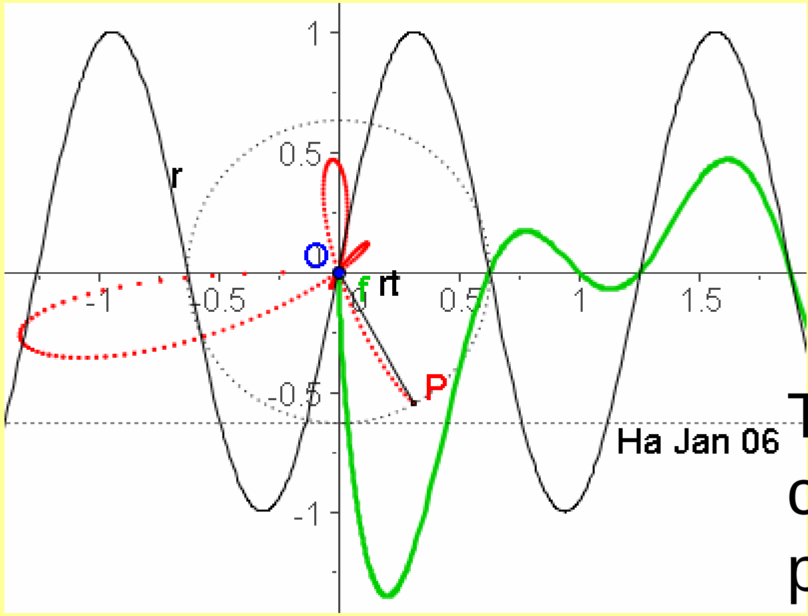
Stating problems and finding solutions



Exercise in the book of Steinberg/ Ebenhöh

$$r(t) = \ln(t)\,\sin(5t)$$

Where does the little extra petal come from?



Wow,
there are two more
even smaller
petals!!!!!

The answer can be found by clever consideration of the polar-cartesian-double-perspective.

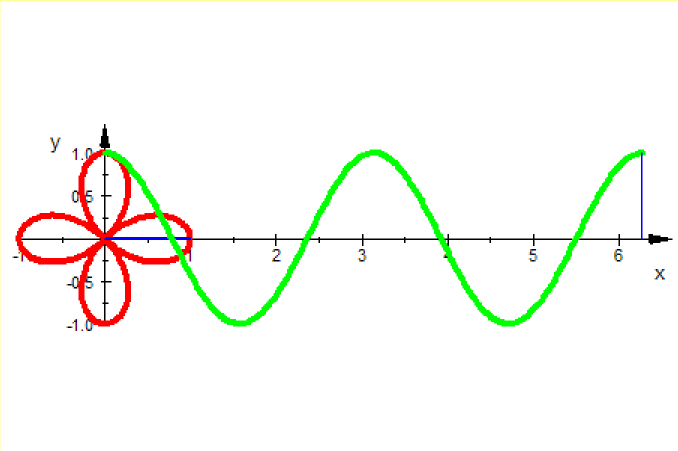
Conclusion

Potentialities for mathematical education

- Consolidation of the concept of a function as an unique (unambiguous) assignment
- The double-perspective of the graphs increases mathematical competence
- Total freedom for students to design classes of curves
- Change of view exhibits the essentials in a better way
- Rich mathematics is best to protect teaching against encrustration in conventionalities

Polar Coordinates in Double-Perspective

by movable and simultaneous visualisation of the respective „cartesian function"



Many thanks!

Danke für Ihre Aufmerksamkeit

You can find all on the internet!

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