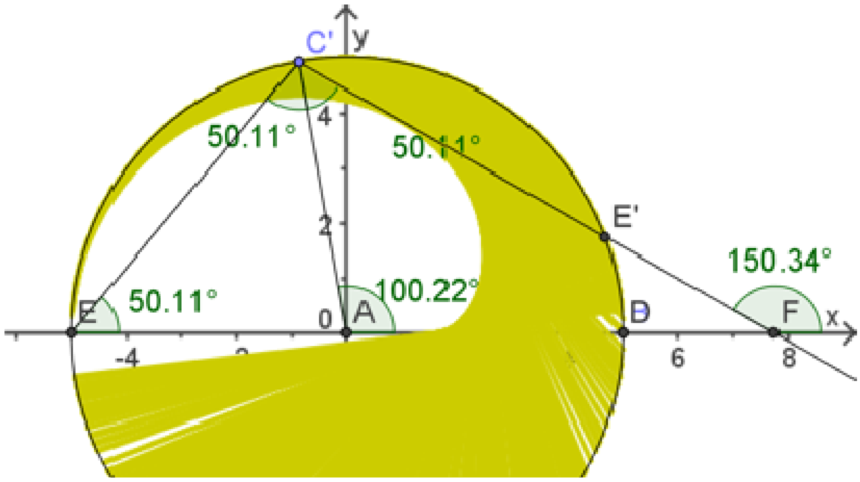
Kardioide als Kaustik



In[16]:= `g[x_, t_] := Tan[3 t] (x - R Cos[2 t]) + R Sin[2 * t];`

Ableitung nach dem Parameter t

In[17]:= `gpunkt = D[g[x, t], t] == 0`

Out[17]= $2 R \operatorname{Cos}[2 t] + 3 (x - R \operatorname{Cos}[2 t]) \operatorname{Sec}[3 t]^2 + 2 R \operatorname{Sin}[2 t] \operatorname{Tan}[3 t] == 0$

In[29]:= `xxg = Solve[gpunkt, x] // TrigExpand // Factor // Simplify`

Out[29]= $\left\{\left\{x \to -\frac{1}{3} R (-2 \operatorname{Cos}[2 t] + \operatorname{Cos}[4 t])\right\}\right\}$

In[34]:= `xxx[t_] := xxg[[1, 1, 2]]; xxx[t]`

Out[34]= $-\frac{1}{3} R (-2 \operatorname{Cos}[2 t] + \operatorname{Cos}[4 t])$

In[35]:= `yyy[t_] := (g[xxx[t], t]); yyy[t] // Simplify`

Out[35]= $\frac{8}{3} R \operatorname{Cos}[t] \operatorname{Sin}[t]^3$

In[44]:= `kreis = ParametricPlot[{R Cos[t], R Sin[t]} /. R → 5, {t, 0, 2 π}];`

In[45]:= `kardi = ParametricPlot[{xxx[t], yyy[t]} /. R → 5, {t, 0, 2 π}];`

In[46]:=

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Show[kreis, kardi]
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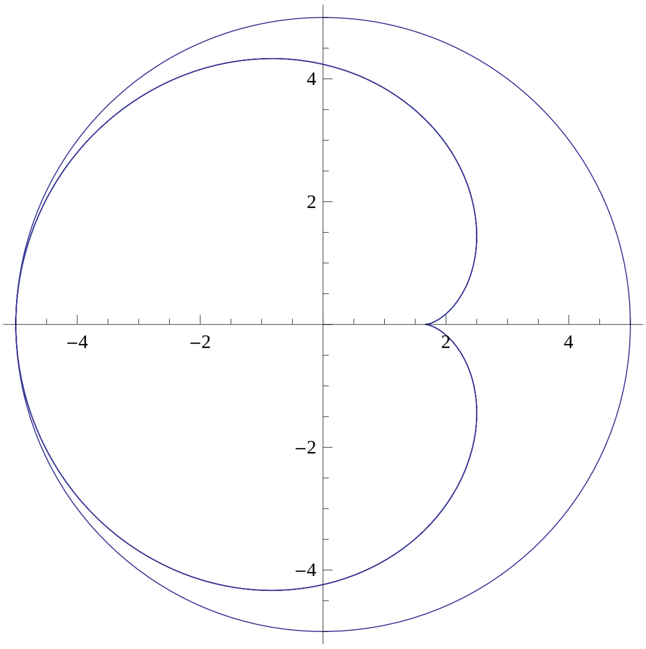
Out[46]=



Eine Explizite Darstellung gelingt nicht

In[26]:= **Eliminate[{x == xxx[t], y == yyy[t]}, {t}] // TrigExpand // Simplify**

Eliminate::ifun :
Inverse functions are being used by Eliminate, so some solutions may not be found; use Reduce for complete solution information. ≫

Out[26]=

$$3x + R\,\mathrm{Cos}[4t] == 2R\,\mathrm{Cos}[2t]\ \&\&$$
$$\mathrm{Cos}[3t]^2 \left(R^4 + 4R^2x^2 + x^4 + 2x^2y^2 + y^4 - 4Rx\left(R^2+x^2+y^2\right)\mathrm{Cos}[2t] + 2R^2\left(x^2+y^2\right)\mathrm{Cos}[4t]\right) ==$$
$$(x - R\,\mathrm{Cos}[2t])^2 \left(R^2 + x^2 + y^2 - 2Rx\,\mathrm{Cos}[2t] + 2Ry\,\mathrm{Sin}[2t]\right)\ \&\&\ \mathrm{Cos}[3t]^2$$
$$\left(3Rx\left(R^2+4x^2\right) - 2\left(6R^2x^2 + 5x^4 + 4x^2y^2 - y^4\right)\mathrm{Cos}[2t] + \left(6Rx - x^2 - y^2\right)\left(x^2+y^2\right)\mathrm{Cos}[4t]\right) ==$$
$$-\frac{1}{2}x\left(x\left(21R^2 + 20x^2 + 8y^2\right)\mathrm{Cos}[2t] + \left(-3R^3 + 2x\left(x^2+y^2\right) - 3R\left(5x^2+y^2\right)\right)\mathrm{Cos}[4t] -\right.$$
$$\left.3\left(-R^2x\,\mathrm{Cos}[6t] + \left(R^2+4x^2\right)y\,\mathrm{Sin}[2t] + R\left(R^2 + 7x^2 + y^2 - 4xy\,\mathrm{Sin}[4t] + Ry\,\mathrm{Sin}[6t]\right)\right)\right)\ \&\&$$
$$\frac{1}{2}\mathrm{Cos}[3t]^2\left(18R^2x^2 + 53x^4 - 14x^2y^2 + 5y^4 - 4\left(18Rx^3 - 2x^4 - x^2y^2 + y^4\right)\mathrm{Cos}[2t] +\right.$$
$$4\left(4x^4 + 5x^2y^2 + y^4\right)\mathrm{Cos}[4t] + 8x^4\,\mathrm{Cos}[6t] + 4x^2y^2\,\mathrm{Cos}[6t] -$$
$$\left.4y^4\,\mathrm{Cos}[6t] + x^4\,\mathrm{Cos}[8t] + 2x^2y^2\,\mathrm{Cos}[8t] + y^4\,\mathrm{Cos}[8t]\right) ==$$
$$\frac{1}{2}x^2\left(9R^2 + 44x^2 + 2y^2 + \left(-63Rx + 8x^2 + 2y^2\right)\mathrm{Cos}[2t] + \left(9R^2 + 25x^2 + y^2\right)\mathrm{Cos}[4t] -\right.$$
$$9Rx\,\mathrm{Cos}[6t] + 8x^2\,\mathrm{Cos}[6t] + 2y^2\,\mathrm{Cos}[6t] + x^2\,\mathrm{Cos}[8t] + y^2\,\mathrm{Cos}[8t] +$$
$$\left.9Ry\,\mathrm{Sin}[2t] + 6xy\,\mathrm{Sin}[2t] - 24xy\,\mathrm{Sin}[4t] + 9Ry\,\mathrm{Sin}[6t] - 6xy\,\mathrm{Sin}[6t]\right)\ \&\&$$
$$(x - R\,\mathrm{Cos}[2t])^2 == \mathrm{Cos}[3t]^2\left(R^2 + x^2 + y^2 - 2Rx\,\mathrm{Cos}[2t] - 2Ry\,\mathrm{Sin}[2t]\right)\ \&\&$$
$$\mathrm{Cos}[3t]^2\left(\left(4x^2 - 2y^2\right)\mathrm{Cos}[2t] + \left(x^2+y^2\right)\mathrm{Cos}[4t] - 3x\,(R - 2y\,\mathrm{Sin}[2t])\right) ==$$
$$\frac{1}{2}x\,(-3R + 8x\,\mathrm{Cos}[2t] + (-3R + 2x)\,\mathrm{Cos}[4t])\ \&\&$$
$$\mathrm{Cos}[3t]^2\left(-2x\left(6R^2 + 5x^2 + 2y^2\right)\mathrm{Cos}[2t] +\right.$$
$$\left.(6R - x)\left(x^2+y^2\right)\mathrm{Cos}[4t] + 3\left(R^3 + 4Rx^2 - Ry^2 + 2y^3\,\mathrm{Sin}[2t]\right)\right) ==$$
$$\frac{1}{2}\left(-x\left(21R^2 + 20x^2\right)\mathrm{Cos}[2t] + \left(3R^3 + 15Rx^2 - 2x^3\right)\mathrm{Cos}[4t] +\right.$$
$$\left.3\left(-R^2x\,\mathrm{Cos}[6t] + \left(R^2+4x^2\right)y\,\mathrm{Sin}[2t] + R\left(R^2 + 7x^2 - 4xy\,\mathrm{Sin}[4t] + Ry\,\mathrm{Sin}[6t]\right)\right)\right)\ \&\&$$
$$2\,\mathrm{Cos}[t]^3\,(1 - 2\,\mathrm{Cos}[2t])^2\,\mathrm{Sin}[t]\left(-R^2 - x^2 - y^2 + 2Rx\,\mathrm{Cos}[2t] + 2Ry\,\mathrm{Sin}[2t]\right) ==$$
$$-(x - R\,\mathrm{Cos}[2t])^2\,\mathrm{Sin}[2t]\ \&\&\ 2\,\mathrm{Cos}[t]^3\,(1 - 2\,\mathrm{Cos}[2t])^2\,\mathrm{Sin}[t]$$
$$\left(\left(4x^2 - 2y^2\right)\mathrm{Cos}[2t] + \left(x^2+y^2\right)\mathrm{Cos}[4t] - 3x\,(R - 2y\,\mathrm{Sin}[2t])\right) ==$$
$$\frac{1}{2}x\,(-3R + 8x\,\mathrm{Cos}[2t] + (-3R + 2x)\,\mathrm{Cos}[4t])\,\mathrm{Sin}[2t]\ \&\&$$
$$2y\,\mathrm{Cos}[2t] == y\,\mathrm{Cos}[4t] + 8x\,\mathrm{Cos}[t]\,\mathrm{Sin}[t]^3\ \&\&$$
$$y + R\,\mathrm{Sec}[3t]\,\mathrm{Sin}[t] == x\,\mathrm{Tan}[3t]\ \&\&$$
$$R^2\,\mathrm{Sin}[4t] + \left(x^2+y^2\right)\mathrm{Sin}[6t] + Rx\,\mathrm{Sin}[8t] ==$$
$$2xy + Ry\,\mathrm{Cos}[4t] + Ry\,\mathrm{Cos}[8t] + Rx\,\mathrm{Sin}[4t] + R^2\,\mathrm{Sin}[6t]\ \&\&$$
$$\mathrm{Cos}[3t]^2\,(-3R + 4x\,\mathrm{Cos}[2t] + x\,\mathrm{Cos}[4t] + 4y\,\mathrm{Sin}[2t] + y\,\mathrm{Sin}[4t]) ==$$
$$-\frac{3R}{2} + 4x\,\mathrm{Cos}[2t] + \left(-\frac{3R}{2} + x\right)\mathrm{Cos}[4t]\ \&\&$$
$$\mathrm{Sin}[3t]\,(y\,\mathrm{Cos}[3t] + R\,\mathrm{Sin}[t] - x\,\mathrm{Sin}[3t]) == 0\ \&\&$$
$$\left(2y\,\mathrm{Cos}[2t] - y\,\mathrm{Cos}[4t] - 8x\,\mathrm{Cos}[t]\,\mathrm{Sin}[t]^3\right)\mathrm{Sin}[6t] == 0\ \&\&$$
$$y\,\mathrm{Cos}[t]\,\mathrm{Cos}[3t] + 6x\,\mathrm{Sin}[2t] + 3R\,\mathrm{Sin}[6t] +$$
$$\mathrm{Cos}[3t]\,(8y\,\mathrm{Cos}[5t] - y\,\mathrm{Cos}[7t] + x\,(3\,\mathrm{Sin}[t] - 8\,\mathrm{Sin}[5t] + \mathrm{Sin}[7t])) == 3R\,\mathrm{Sin}[4t]$$