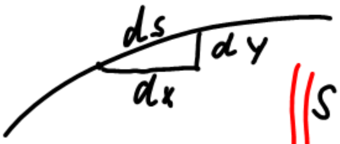
Bogelänge in allen Darstellungen



$$ds^2 = dx^2 + dy^2 = \left(1 + \frac{dy^2}{dx^2}\right) dx^2$$

$$s = \int_{x_0}^{x_1} ds = \int_{x_0}^{x_1} \sqrt{1 + f'(x)^2}\, dx \qquad \text{kartesisch}$$

Parametrisch $\quad x = x(t) \quad y = y(t) \qquad \dot{x} = \frac{dx}{dt} \quad \dot{y} = \frac{dy}{dt} \qquad ds^2 = \left(\frac{dx^2}{dt^2} + \frac{dy^2}{dt^2}\right) dt^2$

$$s = \int_{t_0}^{t_1} \sqrt{\dot{x}^2 + \dot{y}^2}\, dt$$

Polar $\quad r = r(\varphi) \qquad x = r(\varphi)\cos\varphi \quad y = r(\varphi)\sin\varphi \qquad \dot{x} = \dot{r}\cos\varphi - r\sin\varphi \quad \dot{y} = \dot{r}\sin\varphi + r\cos\varphi$

$$\begin{aligned}\dot{x}^2 + \dot{y}^2 &= \dot{r}^2\cos^2\varphi - 2\dot{r}r\cos\varphi\sin\varphi + r^2\sin^2\varphi \\ &\quad + \dot{r}^2\sin^2\varphi + 2\dot{r}r\sin\varphi\cos\varphi + r^2\sin^2\varphi \\ &= \dot{r}^2 \quad + \quad 0 \quad + \quad r^2\end{aligned}$$

$$s = \int_{\varphi_0}^{\varphi_1} \sqrt{\dot{r}^2 + r^2}\, d\varphi$$

Beispiele $\quad$ Parabel $\quad f(x) = x^2 \quad f'(x) = 2x$

$$s = \int_0^b \sqrt{1 + 4x^2}\, dx = \left[\frac{\operatorname{asinh}(2x)}{4} + \frac{x\sqrt{4x^2+1}}{2}\right]_0^b = \frac{1}{4}\left(\sinh^{-1}(2b) + 2b\sqrt{4b^2+1}\right) \underset{b=\sqrt{2}}{=} \frac{1}{4}\left(\sinh^{-1}(\sqrt{8}) + 3\sqrt{8}\right)$$

Archimedische Spirale $\quad r(\varphi) = \varphi \quad \dot{r}(\varphi) = 1$

$$s = \int_0^{2\pi} \sqrt{1 + \varphi^2}\, d\varphi = \left[\frac{1}{2}\left(\sinh^{-1}(\varphi) + \varphi\sqrt{1+\varphi^2}\right)\right]_0^{2\pi} \approx 21{,}2563$$

neill(x):=x^(3/2);

$\text{neill}(x) := x^{3/2}$

integrate(sqrt(1+diff(neill(x),x,1)^2), x);

$$\frac{8\left(\frac{9x}{4}+1\right)^{3/2}}{27}$$

$\text{nx}(t) := t^2$

$\text{ny}(t) := t^3$

integrate(sqrt(diff(nx(t),t,1)^2+diff(ny(t),t,1)^2), t);

$$\int \sqrt{9t^4 + 4t^2}\, dt$$

integrate(sqrt(diff(nx(t),t,1)^2+diff(ny(t),t,1)^2), t,0,2/sqrt(3));

$$\frac{56}{27}$$