Übungen Vollst. Induktion
ggf. rekursive Formel → explizite Formel

$$\sum_{i=1}^{n} i^2 = 1 + 2^2 + 3^2 + \dots n^2 = 1+4+9+16+\dots+n^2$$

Summe der Quadratzahlen

Hoffmann 09

quadratzahlen.pdf

Formel selbst entwickeln:
Vermutung Polynom 3. Grades
⇒ aus vier Werten herstellen

$a_1 = 1$ $\quad p(x) = ax^3 + bx^2 + cx + d$

$a_2 = 5$ $\quad 1 = a + b + c + d$

$a_3 = 14$ $\quad 5 = 8a + 4b + 2c + d$

$a_4 = 30$ $\quad 14 = 27a + 9b + 3c + d$

$\quad 30 = 64a + 16b + 4c + d$

$a = \frac{1}{3} \quad b = \frac{1}{2} \quad c = \frac{1}{6} \quad d = 0$

$$a_n = \frac{1}{6}\left(2n^3 + 3n^2 + n\right)$$

$$\boxed{a_n = \frac{1}{6} \cdot n \cdot (n+1)(2n+1)}$$

$2n^2 + 3n + 1 = 0$

$n^2 + \frac{3}{2}n + \left(\frac{3}{4}\right)^2 = -\frac{1}{2} + \frac{9}{16}$

$n = -\frac{3}{4} \pm \frac{1}{4}$

$n = -1 \lor n = -\frac{1}{2}$

$(n+1)\left(n+\frac{1}{2}\right) \cdot 2$

$(n+1)(2n+1)$

IV klar bei dem Ansatz

IA $\quad a_n = \frac{1}{6} n(n+1)(2n+1)$

$n \rightsquigarrow n+1$

Ziel $a_{n+1} = \frac{1}{6}(n+1)(n+2)(2(n+1)+1)$

Rekursion sicher $\quad a_{n+1} = a_n + (n+1)^2$

$$a_{n+1} = a_n + (n+1)^2 = \frac{1}{6}\left(n(n+1)(2n+1) + 6(n+1)^2\right)$$

$$= \frac{1}{6}(n+1)\left(2n^2 + n + 6n + 6\right) = \frac{1}{6}(n+1)(2n^2 + 7n + 6) = *$$

NR $\quad (n+2)(2n+3) = 2n^2 + 3n + 4n + 6 = 2n^2 + 7n + 6$

$$* = \frac{1}{6}(n+1)(n+2)(2n+3) \qquad \underline{\text{qed}}$$