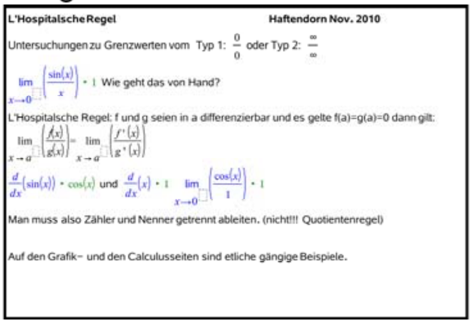
Einige Grenzwerte



L'Hospitalsche Regel **Haftendorn Nov. 2010**

Untersuchungen zu Grenzwerten vom Typ 1: $\frac{0}{0}$ oder Typ 2: $\frac{\infty}{\infty}$

$\lim\limits_{x\to 0}\left(\frac{\sin(x)}{x}\right) \blacktriangleright 1$ Wie geht das von Hand?

L'Hospitalsche Regel: f und g seien in a differenzierbar und es gelte f(a)=g(a)=0 dann gilt:

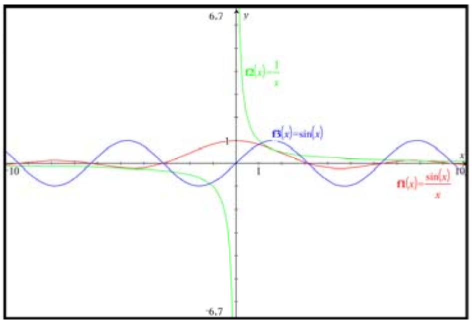
$\lim\limits_{x\to a}\left(\frac{f(x)}{g(x)}\right)=\lim\limits_{x\to a}\left(\frac{f'(x)}{g'(x)}\right)$

$\frac{d}{dx}(\sin(x)) \blacktriangleright \cos(x)$ und $\frac{d}{dx}(x) \blacktriangleright 1$ $\lim\limits_{x\to 0}\left(\frac{\cos(x)}{1}\right) \blacktriangleright 1$

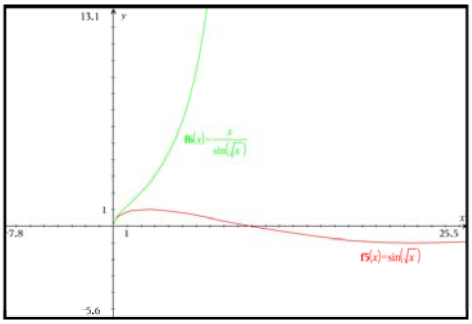
Man muss also Zähler und Nenner getrennt ableiten. (nicht!!! Quotientenregel)

Auf den Grafik- und den Calculusseiten sind etliche gängige Beispiele.

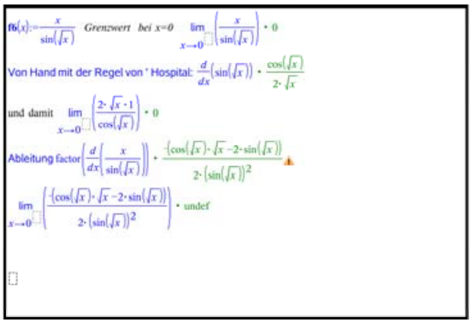
1.1



1.2



1.3



$f6(x):=\frac{x}{\sin(\sqrt{x})}$ *Grenzwert bei x=0* $\lim\limits_{x\to 0}\left(\frac{x}{\sin(\sqrt{x})}\right) \blacktriangleright 0$

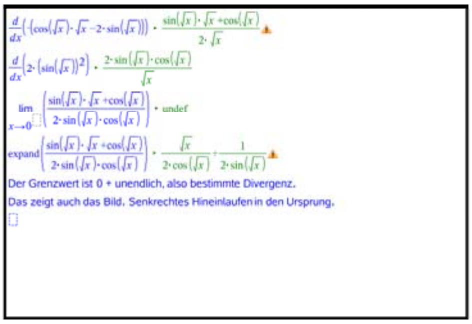
Von Hand mit der Regel von ' Hospital: $\frac{d}{dx}(\sin(\sqrt{x})) \blacktriangleright \frac{\cos(\sqrt{x})}{2\cdot\sqrt{x}}$

und damit $\lim\limits_{x\to 0}\left(\frac{2\cdot\sqrt{x}\cdot 1}{\cos(\sqrt{x})}\right) \blacktriangleright 0$

Ableitung $\text{factor}\left(\frac{d}{dx}\left(\frac{x}{\sin(\sqrt{x})}\right)\right) \blacktriangleright \frac{-(\cos(\sqrt{x})\cdot\sqrt{x}-2\cdot\sin(\sqrt{x}))}{2\cdot(\sin(\sqrt{x}))^2}$

$\lim\limits_{x\to 0}\left(\frac{-(\cos(\sqrt{x})\cdot\sqrt{x}-2\cdot\sin(\sqrt{x}))}{2\cdot(\sin(\sqrt{x}))^2}\right) \blacktriangleright \text{undef}$

1.4



$\frac{d}{dx}(-(\cos(\sqrt{x})\cdot\sqrt{x}-2\cdot\sin(\sqrt{x}))) \blacktriangleright \frac{\sin(\sqrt{x})\cdot\sqrt{x}+\cos(\sqrt{x})}{2\cdot\sqrt{x}}$

$\frac{d}{dx}(2\cdot(\sin(\sqrt{x}))^2) \blacktriangleright \frac{2\cdot\sin(\sqrt{x})\cdot\cos(\sqrt{x})}{\sqrt{x}}$

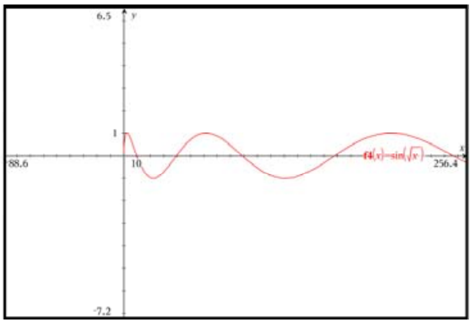
$\lim\limits_{x\to 0}\left(\frac{\sin(\sqrt{x})\cdot\sqrt{x}+\cos(\sqrt{x})}{2\cdot\sin(\sqrt{x})\cdot\cos(\sqrt{x})}\right) \blacktriangleright \text{undef}$

$\text{expand}\left(\frac{\sin(\sqrt{x})\cdot\sqrt{x}+\cos(\sqrt{x})}{2\cdot\sin(\sqrt{x})\cdot\cos(\sqrt{x})}\right) \blacktriangleright \frac{\sqrt{x}}{2\cdot\cos(\sqrt{x})}+\frac{1}{2\cdot\sin(\sqrt{x})}$

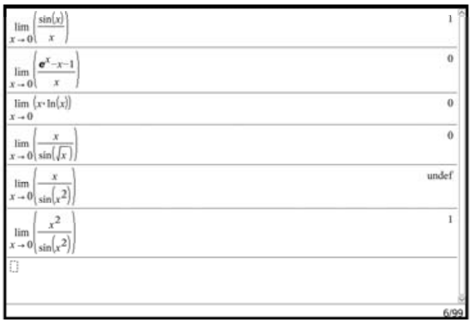
Der Grenzwert ist 0 + unendlich, also bestimmte Divergenz.

Das zeigt auch das Bild. Senkrechtes Hineinlaufen in den Ursprung.

1.5



1.6



$\lim\limits_{x\to 0}\left(\frac{\sin(x)}{x}\right)$	1
$\lim\limits_{x\to 0}\left(\frac{e^x-x-1}{x}\right)$	0
$\lim\limits_{x\to 0}(x\cdot\ln(x))$	0
$\lim\limits_{x\to 0}\left(\frac{x}{\sin(\sqrt{x})}\right)$	0
$\lim\limits_{x\to 0}\left(\frac{x}{\sin(x^2)}\right)$	undef
$\lim\limits_{x\to 0}\left(\frac{x^2}{\sin(x^2)}\right)$	1

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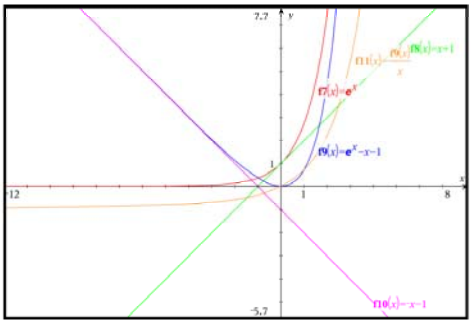
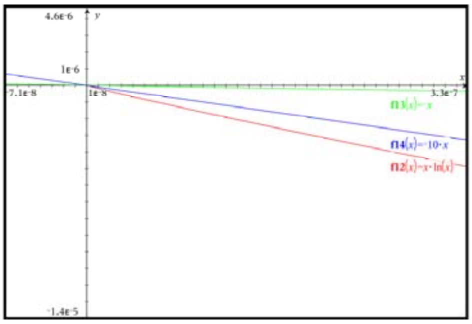
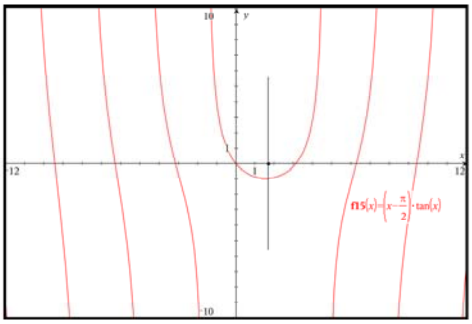
1.7



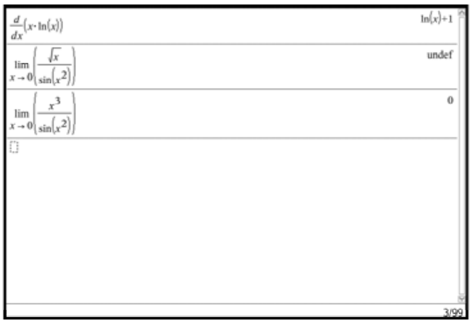
1.8


4.6E-6
y
1E-6
x
-7.1E-8
1E-8
3.3E-7
$f3(x)=-x$
$f4(x)=-10\cdot x$
$f2(x)=x\cdot\ln(x)$
-1.4E-5

1.9


10
y
x
-12
1
12
-10
$f5(x)=\left(x-\frac{\pi}{2}\right)\cdot\tan(x)$

1.10


$\frac{d}{dx}(x\cdot\ln(x))$ $\ln(x)+1$
$\lim_{x\to 0}\left(\frac{\sqrt{x}}{\sin(x^2)}\right)$ undef
$\lim_{x\to 0}\left(\frac{x^3}{\sin(x^2)}\right)$ 0
3/99

1.11