Differentialquotienten

Differenzenquotient = Sekantensteigung Ha S.66
in einem Punkt P der Funktion f

$$\frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{h} = m_{sek}(h) \qquad f: x \to y = f(x)$$

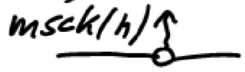
$$\Big\downarrow h \to 0 \qquad\qquad \Big\downarrow h \to 0 \qquad\qquad \Big\downarrow h \to 0$$

falls existiert

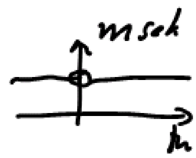
$$\frac{dy}{dx} = f'(x) = m_{tang}(x)$$

Differentialquotient von $y = f(x)$ = Ableitung von f = Tangentensteigung von f in $P(x,y)$

$$f(x) = 1 \qquad \frac{\Delta y}{\Delta x} = \frac{1-1}{h} = \frac{0}{h} \underset{\substack{\uparrow \\ h \neq 0}}{=} 0 \xrightarrow[h \to 0]{} 0$$

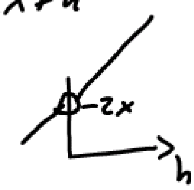


$$f(x) = x \qquad \frac{\Delta y}{\Delta x} = \frac{x+h-x}{h} = \frac{h}{h} \underset{\substack{\uparrow \\ h \neq 0}}{=} 1 \xrightarrow[h \to 0]{} 1$$



$$f(x) = x^2 \qquad \frac{\Delta y}{\Delta x} = \frac{(x+h)^2 - x^2}{h} = \frac{2xh + h^2}{h} = \frac{h(2x+h)}{h} \underset{h \neq 0}{=} 2x + h$$

$$\lim_{h \to 0} (2x + h) = 2x$$



$f(x) = x^3$ ...... ebenso $f'(x) = 3x^2$

$$k \in \mathbb{N} \qquad (x^k)' = k\,x^{k-1}$$

$$k = 0 \qquad (1)' = 0$$

$f(x)=\frac{1}{x}$ $\quad \frac{\frac{1}{x+h}-\frac{1}{x}}{h}=\frac{x-(x+h)}{x(x+h)\cdot h}=\frac{-h}{x(x+h)h}$

$\left(\frac{1}{x}\right)'=-\frac{1}{x^2}$ $\quad \underset{h\neq 0}{\overset{\uparrow}{=}}\frac{-1}{x(x+h)} \xrightarrow{h\to 0} \frac{-1}{x^2}$

$f(x)=\sqrt{x}$ $\quad \frac{\sqrt{x+h}-\sqrt{x}}{h}=\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

$=\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}=\frac{h}{h(\sqrt{x+h}+\sqrt{x})}\underset{h\neq 0}{\overset{\uparrow}{=}}\frac{1}{\sqrt{x+h}+\sqrt{x}}\xrightarrow{h\to 0}\frac{1}{2\sqrt{x}}$

$(\sqrt{x})'=\frac{1}{2\sqrt{x}}$

$(ag)'=ag'$,

$f(x)=ag(x)$, $a\in\mathbb{R}$ $\quad \frac{a(g(x+h)-ag(x)}{h}=a\frac{g(x+h)-g(x)}{h}\xrightarrow{h\to 0}a\,g'(x)$ falls existiert

$f(x)=g(x)+f(x)$ $\quad \frac{g(x+h)+f(x+h)-g(x)+f(x)}{h}=\frac{g(x+h)-g(x)}{h}+\frac{f(x+h)-f(x)}{h}$

$\downarrow h\to 0 \qquad \downarrow h\to 0$

$g'(x)+f'(x)$ falls ex.

$(g+f)'=g'+f'$

Linearität des Differentialoperators

$$(af+bg)'=af'+bg'$$

$$\left(5x^2-3x+\frac{7}{x}\right)'=10x-3-\frac{7}{x^2}$$

Verschieben: $\left(f(x-a)+b\right)'=f'(x-a)$

$$\left(\sqrt{x-11}+3\right)'=\frac{1}{2\sqrt{x-11}}$$

Strecken $\longleftrightarrow$ $f(x)=g(ax)$ Alle Steigungsdreiecke haben $\frac{1}{a}$-fache Breite bei gleicher Höhe daher wird die Steigung a-fach

$g'(ax)=a\,g'(ax)$

$$\left(\sqrt{7x+4}\right)'=7\cdot\frac{1}{2\sqrt{7x+4}}$$