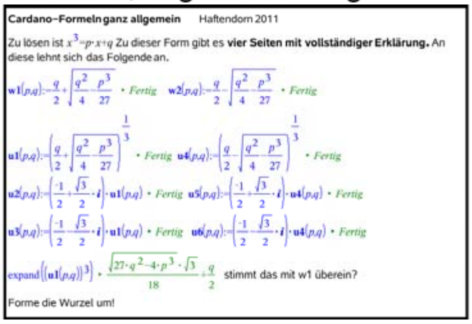
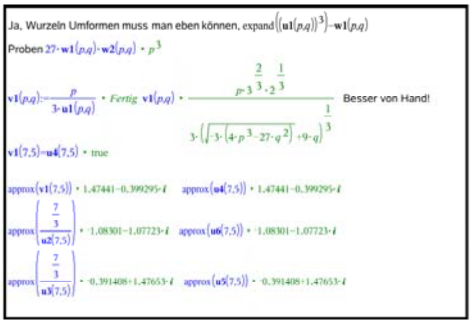
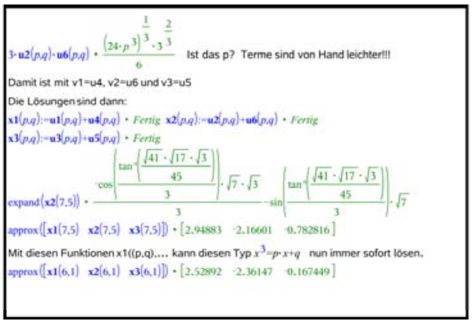
Cardano, allgem. Lösungsfunk…



1.1



1.2



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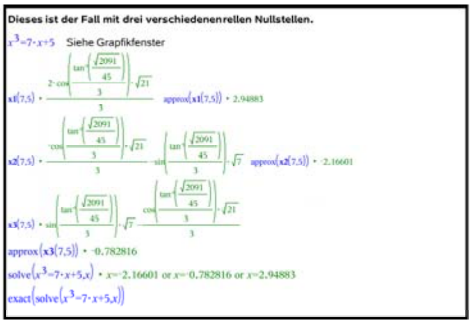

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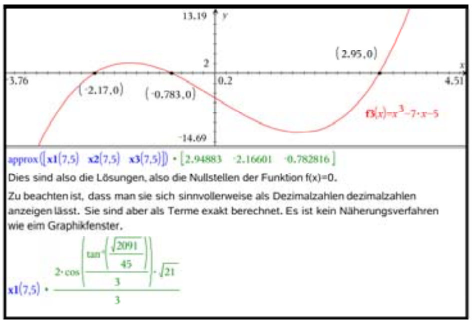

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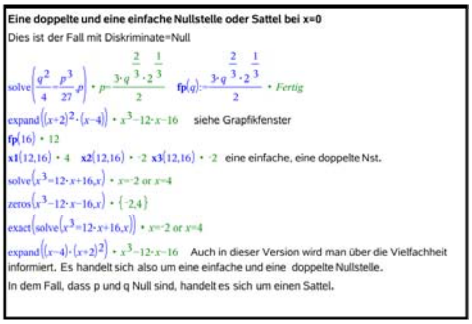

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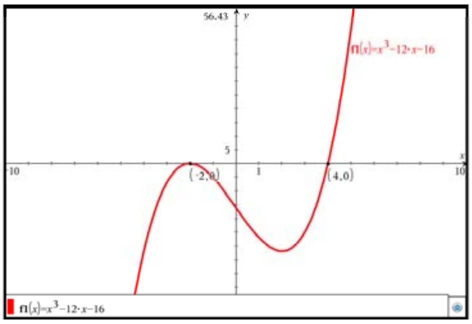

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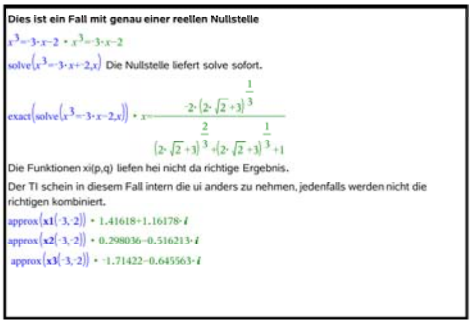

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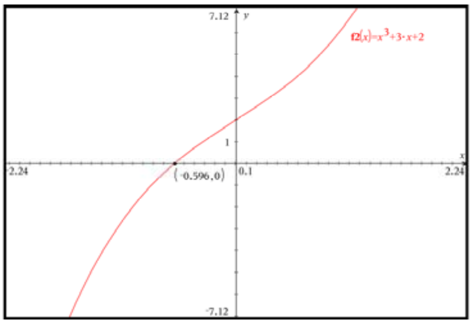
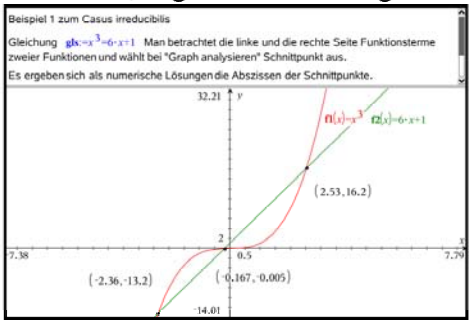


1.9

Cardano, eigene Gleichungen

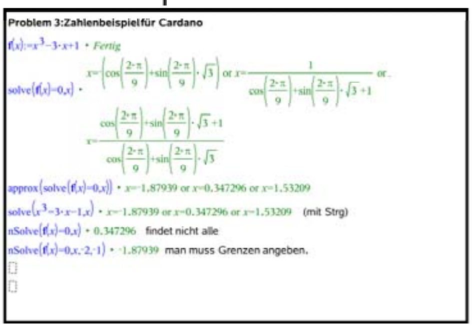


2.1

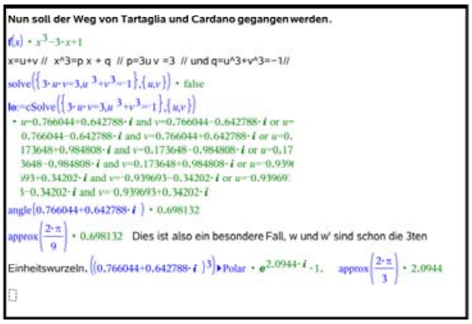
2.2

2.3

Zahlenbeispiel



3.1



3.2

$\text{cSolve}\left(\frac{1}{u^3}+u^3=1,u\right)$ ▸ $u=0.766044+0.642788\cdot i$ or $u=0.766044-0.642788\cdot i$ or $u=0.173648+0.984808\cdot i$ or $u=0.173648-0.984808\cdot i$ or $u=-0.939693+0.34202\cdot i$ or $u=-0.939693-0.34202\cdot i$

Dies sind die erwarteten sechs Lösungen, von denen man die passenden, die p=3 u v erfüllen addieren muss, um die reellen Lösungen zu erhalten.

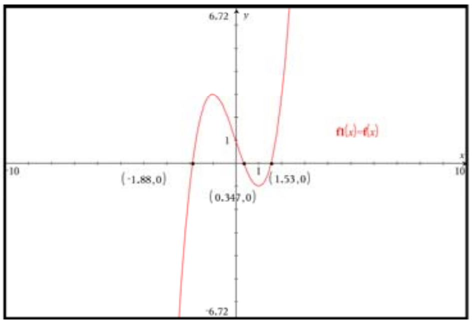
$\text{solve}(f(x)=0,x)$ ▸ $x=-1.87939$ or $x=0.347296$ or $x=1.53209$

$0.766044+0.642788\cdot i+0.766044-0.642788\cdot i$ ▸ 1.53209

$0.173648+0.984808\cdot i+0.173648-0.984808\cdot i$ ▸ 0.347296

$-0.939693+0.34202\cdot i+-0.939693-0.34202\cdot i$ ▸ -1.87939

3.3



3.4

Allg.Poynom 3 Grades anpas…

Problem 3 Allgemeine Polynom 3. Grades anpassen

Die Cardano-Formeln aus Problem 1 erfordern $x^3-p\,x-q=0$

Anpassung von Polynomen 3. Grades mit quadratischen Term an diese Form:

$\text{fallg}(x):=x^3+a\cdot x^2+b\cdot x+c$ ▸ *Fertig*

Man muss den Wendepunkt waagerecht auf die y-Achse verschieben:

$\text{xwallg}:=\text{zeros}\left(\frac{d}{dx}\left(\frac{d}{dx}(\text{fallg}(x))\right),x\right)[1]$ ▸ $\frac{-a}{3}$

$\text{fcard}(x):=\text{fallg}(x+\text{xwallg})$ ▸ *Fertig*

$\text{fcard}(x)$ ▸ $x^3-\frac{(a^2-3\cdot b)\cdot x}{3}+\frac{2\cdot a^3}{27}-\frac{a\cdot b}{3}+c$

4.1

Die Cardano-Formeln aus Problem 1 erfordern $x^3-p\,x-q=0$

Anpassung von Polynomen 3. Grades mit quadratischen Term an diese Form:

$f(x):=(x+2)\cdot(x-1)\cdot(x-2)$ ▸ *Fertig* $\text{expand}(f(x))$ ▸ $x^3-x^2-4\cdot x+4$

Man muss den Wendepunkt waagerecht auf die y-Achse verschieben:

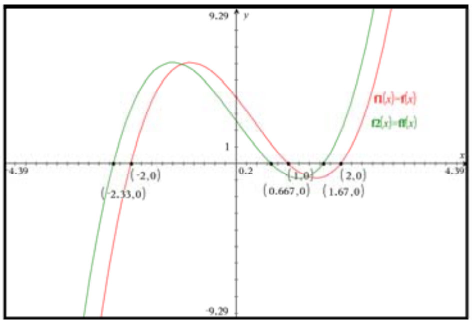
$\text{xw}:=\text{zeros}\left(\frac{d}{dx}\left(\frac{d}{dx}(f(x))\right),x\right)[1]$ ▸ $\frac{1}{3}$

$\text{ff}(x):=f(x+\text{xw})$ ▸ *Fertig* $\text{expand}(\text{ff}(x))$ ▸ $x^3-\frac{13\cdot x}{3}+\frac{70}{27}$

Also ist für die Cardano-Formeln $p=\frac{13}{3}$ und $q=\frac{70}{27}$

Siehe Graph-Fenster

4.2



4.3

Nochmal x^2 eliminieren

Problem 4 Anpassung Sonderfall: kein x-Term

Die Cardano-Formeln aus Problem 1 erfordern $x^3-p\,x-q=0$

Anpassung von Polynomen 3. Grades mit quadratischen Term an diese Form:

$f(x):=x^3+b\cdot x^2+c$ ▸ *Fertig* und nun $\text{ff}(x):=\text{expand}\left(f\left(x-\frac{b}{3}\right)\right)$ ▸ *Fertig*

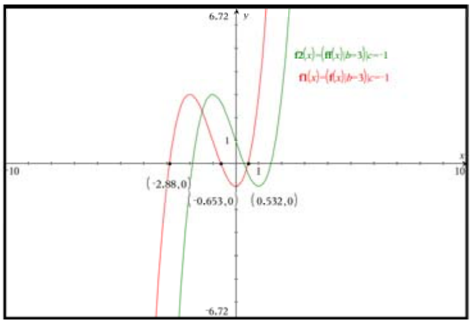
$\text{ff}(x)$ ▸ $x^3-\frac{b^2\cdot x}{3}+\frac{2\cdot b^3}{27}+c$

Wenn man den Zusammenhang mit der Wendepunktlage kennt, überrascht diese Vorschlag der Formelsammlungen nicht.

$\text{approx}(\text{cSolve}((f(x)=0|b=3)|c=-1,x))$ ▸ $x=-2.87939$ or $x=-0.652704$ or $x=0.532089$

$\text{approx}(\text{cSolve}((\text{ff}(x)=0|b=3)|c=-1,x))$ ▸ $x=-1.87939$ or $x=0.347296$ or $x=1.53209$ um 1 verschoben:

5.1



5.2