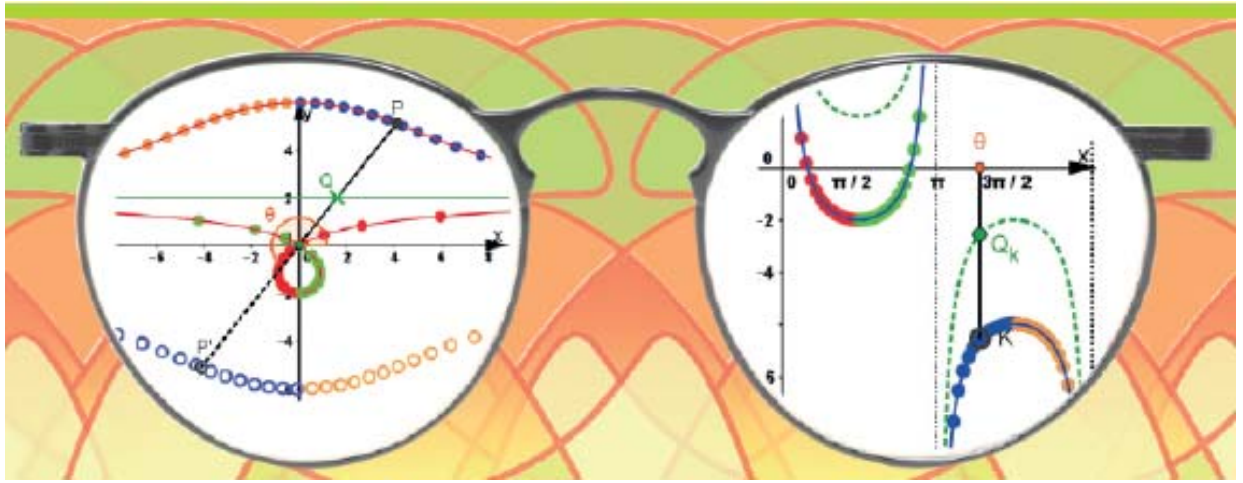
Kurven verstehen durch zwei Perspektiven



Vortrag für die Berliner Mathematische Gesellschaft

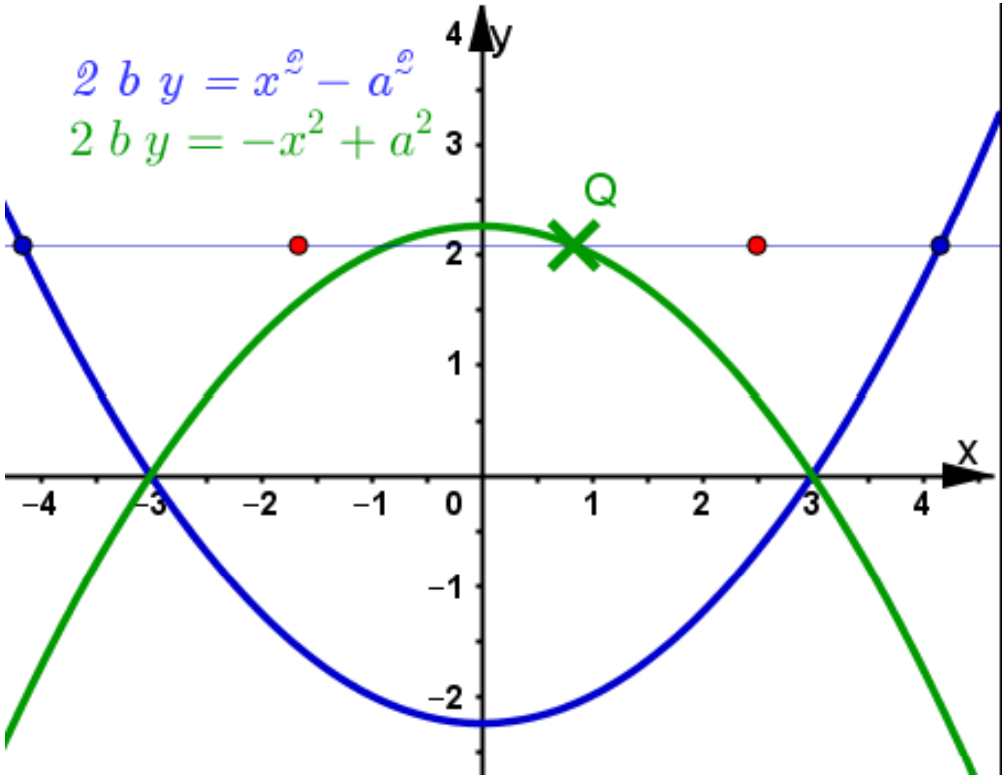
am 8. Februar 2018

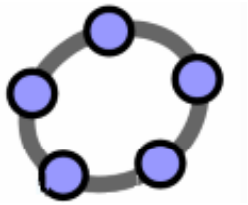
Mittenkurven

Zwei „harmlose“ Parabeln,

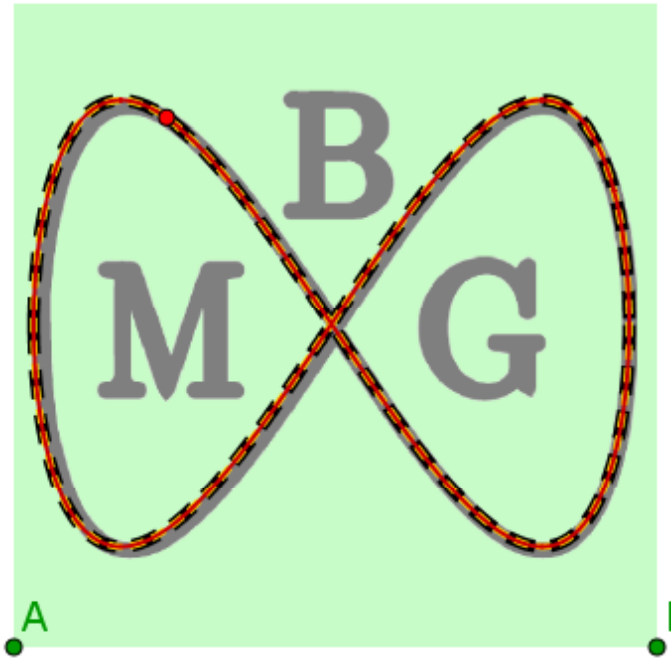
Q wandert,

die Mittelpunkte werden dadurch bewegt.

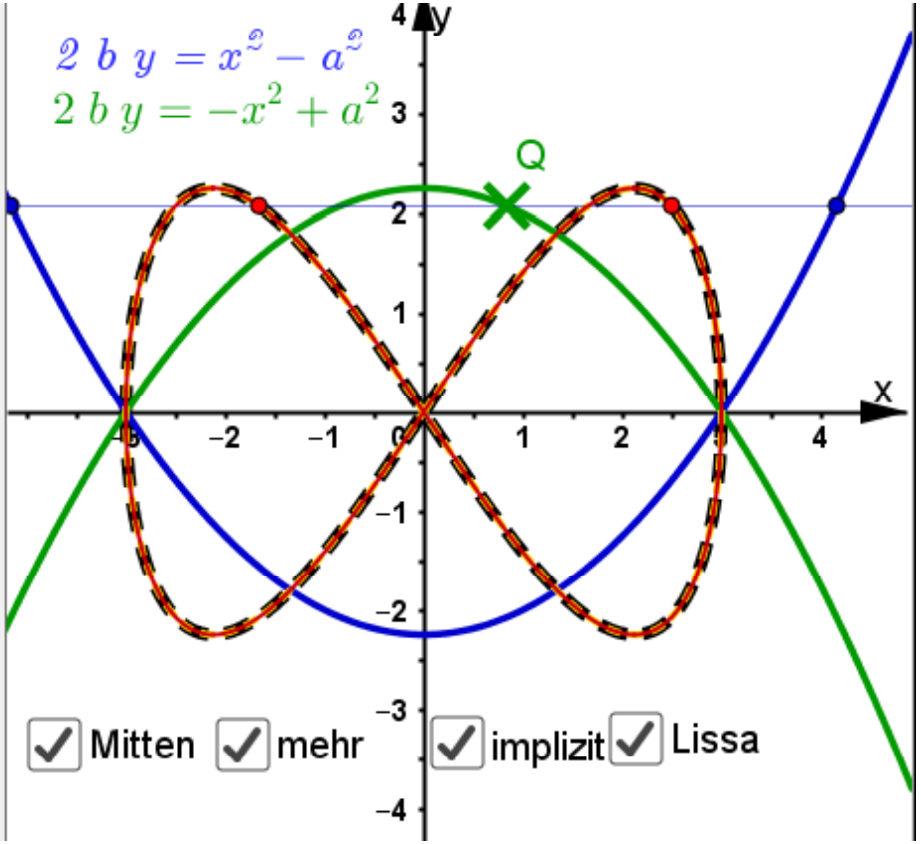




Mittenkurven



explizite Gleichung $2^2\, y^2 = 3^2\, x^2 - x^4$

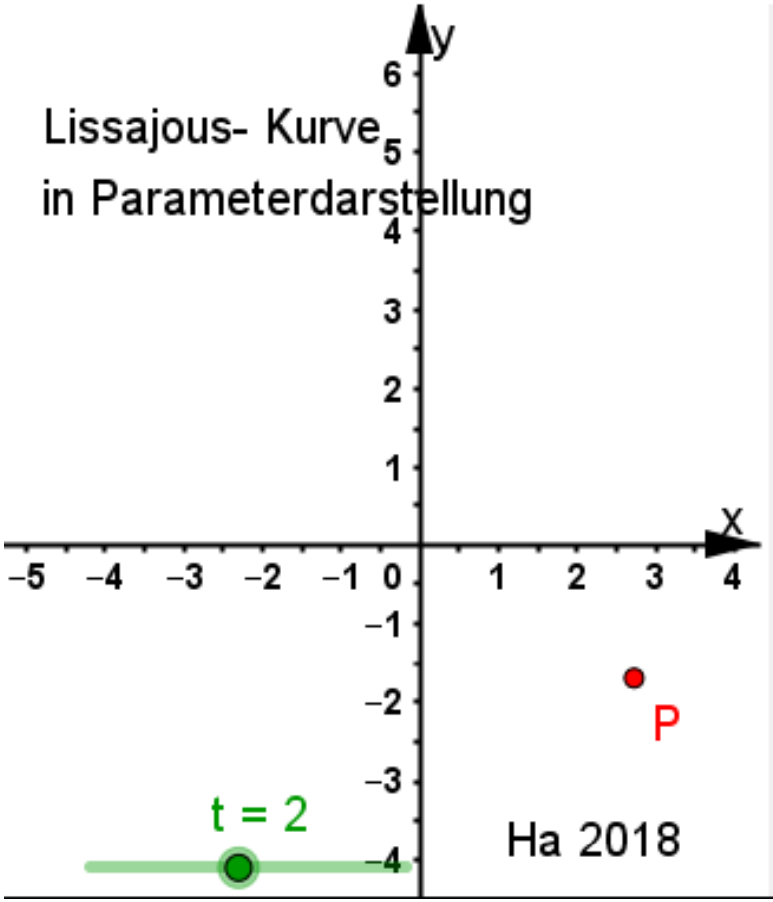


Gerono‘sche Lemniskate

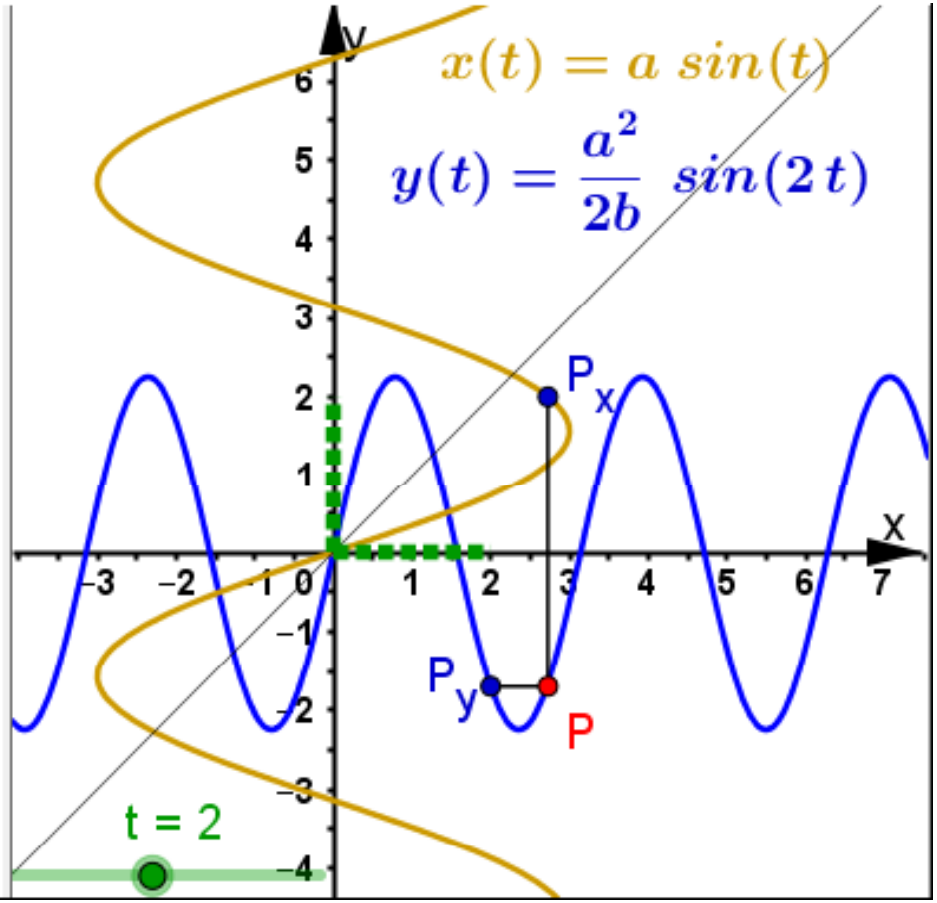
Camille-Christophe Gerono (* 1799 in Paris; † 1891)

Parameterkurven

Lissajous-Kurve

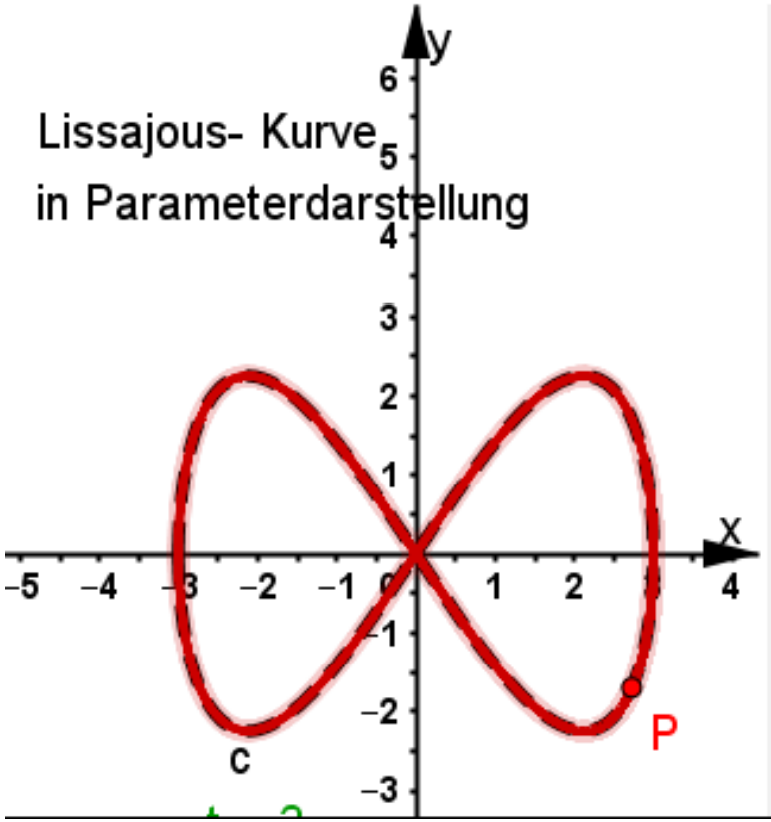


Schwingendes Pendel

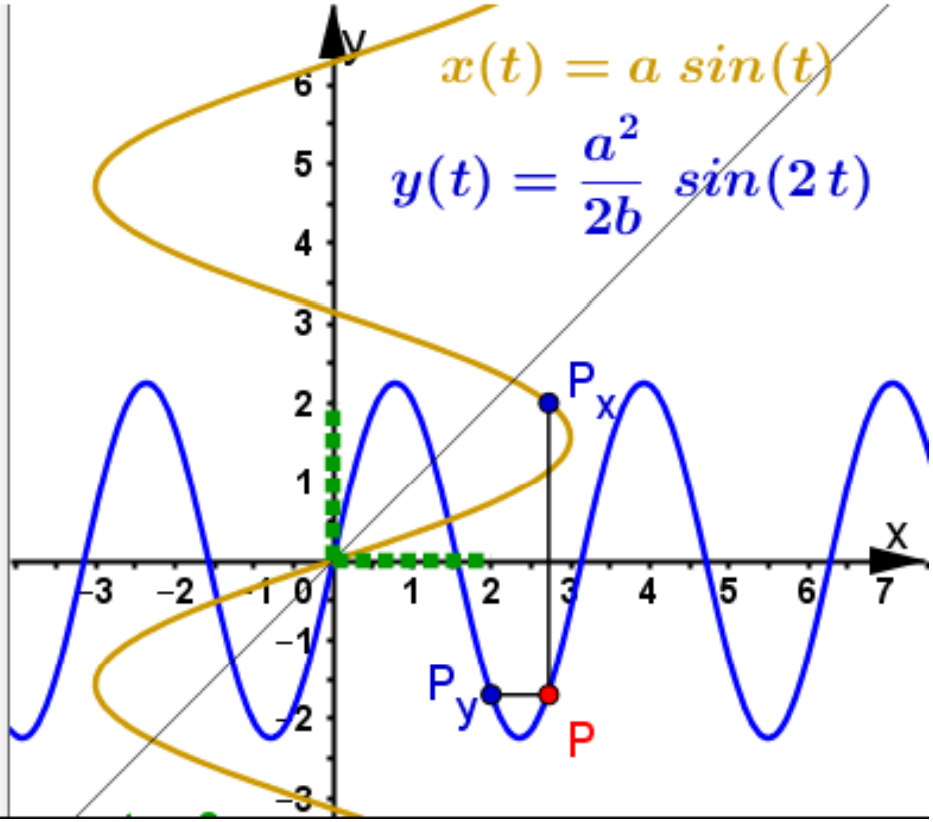


Parameterkurven

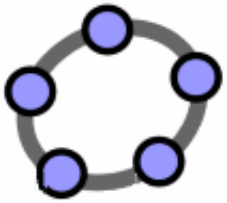
Lissajous-Kurve



Schwingendes Pendel



Polarkurven



Polardarstellung

Zum Polarwinkel θ
gehört der Polarradius $r(\theta)$

$$b^2 y^2 = a^2 x^2 - x^4$$

Gerono‘sche Lemniskate

$$x(\theta) = r\cos(\theta)$$

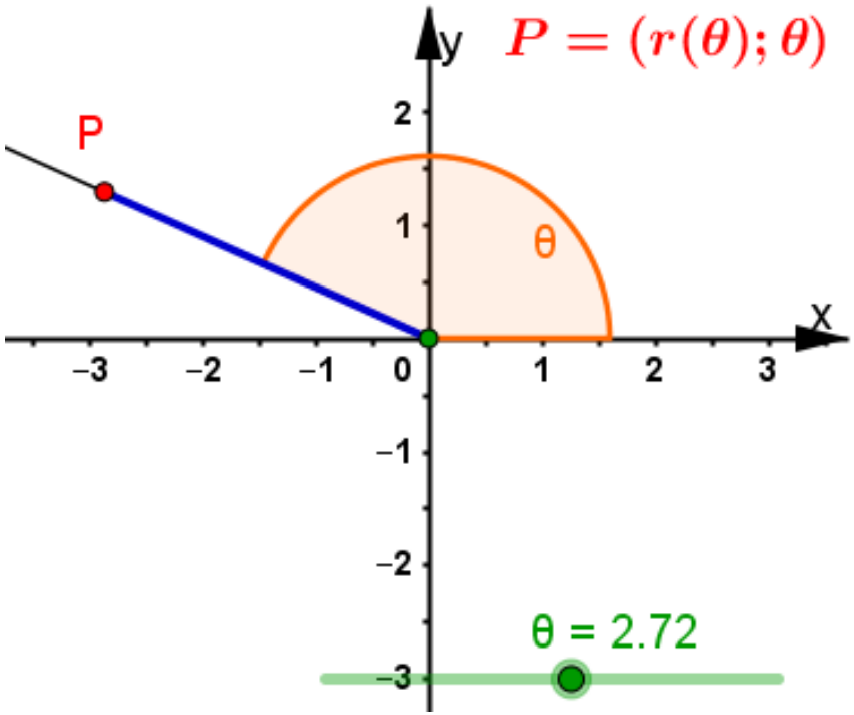
$$y(\theta) = r\sin(\theta)$$

$$\Rightarrow \quad r(\theta) = \frac{\sqrt{a^2\cos(\theta)^2 - b^2\sin(\theta)^2}}{\cos(\theta)^2}$$

Polarkurven

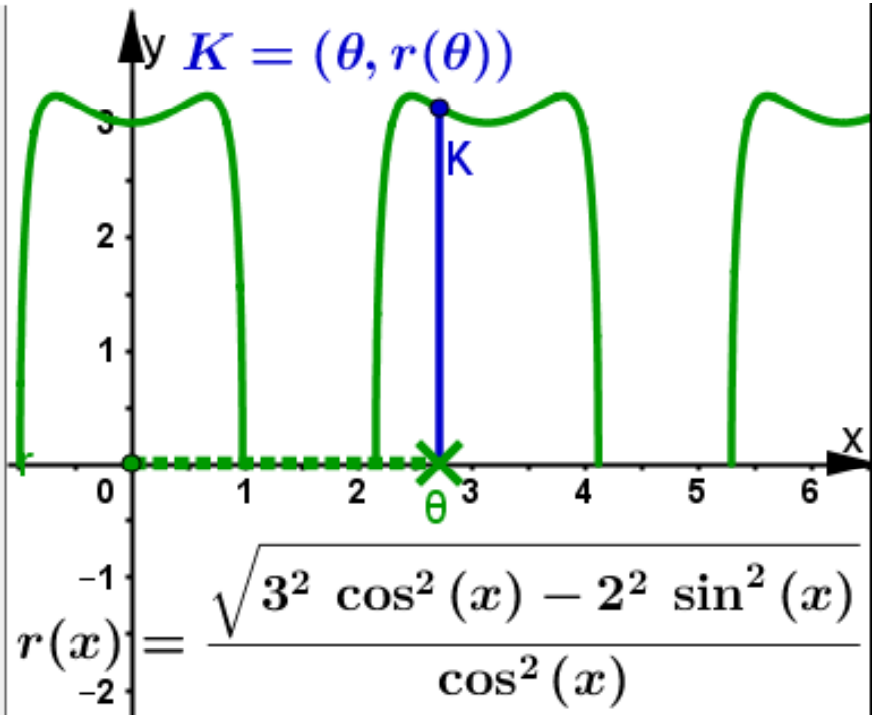
Polardarstellung

Zum Polarwinkel θ
gehört der Polarradius $r(\theta)$



kartesische Darstellung

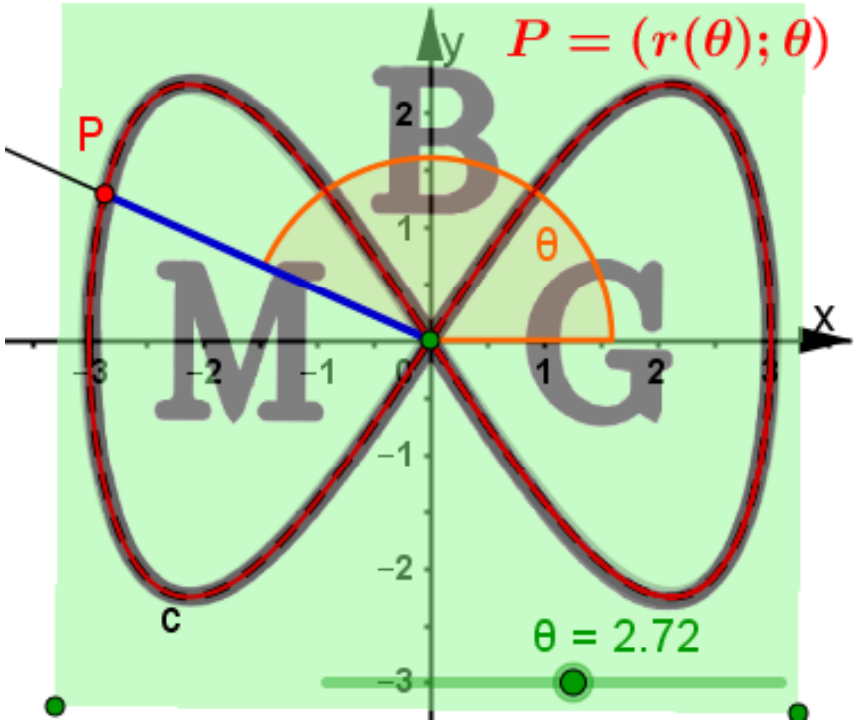
Zur Abszisse θ
gehört die Ordinate $r(\theta)$



Polarkurven

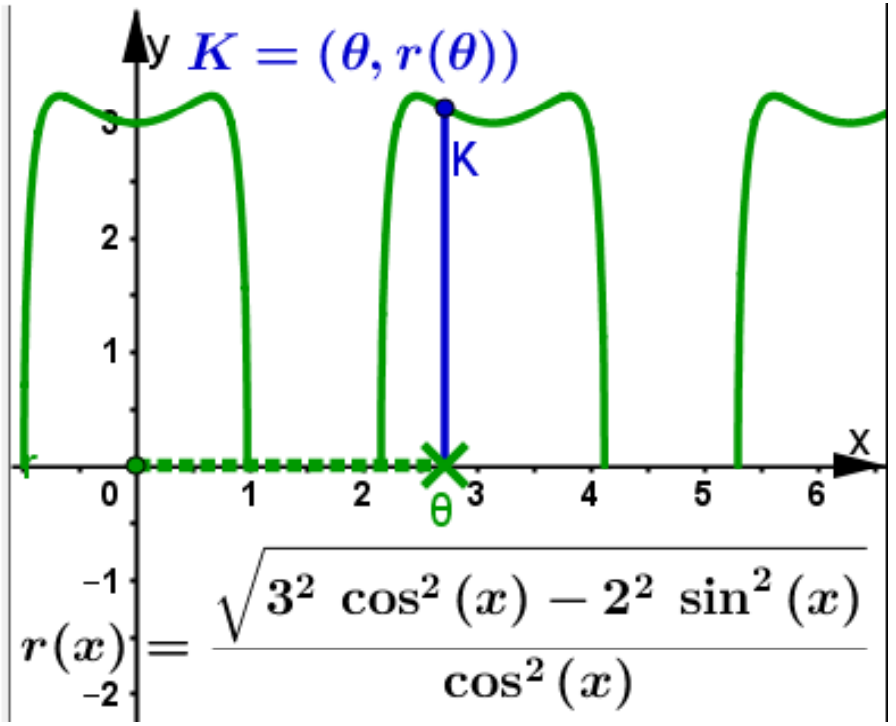
Polardarstellung

Zum Polarwinkel θ
gehört der Polarradius $r(\theta)$

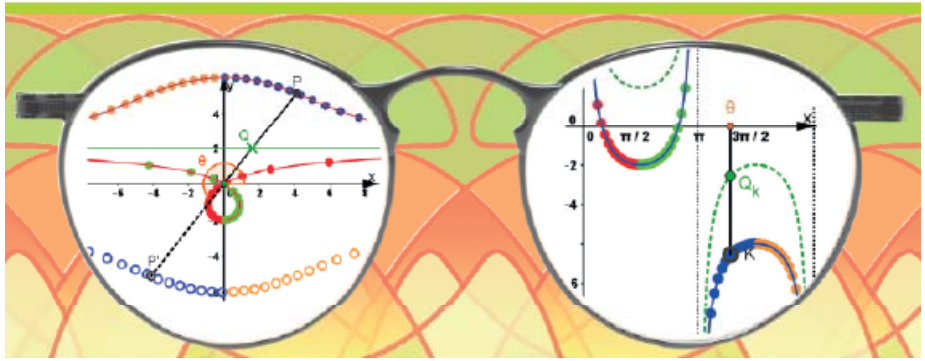


kartesische Darstellung

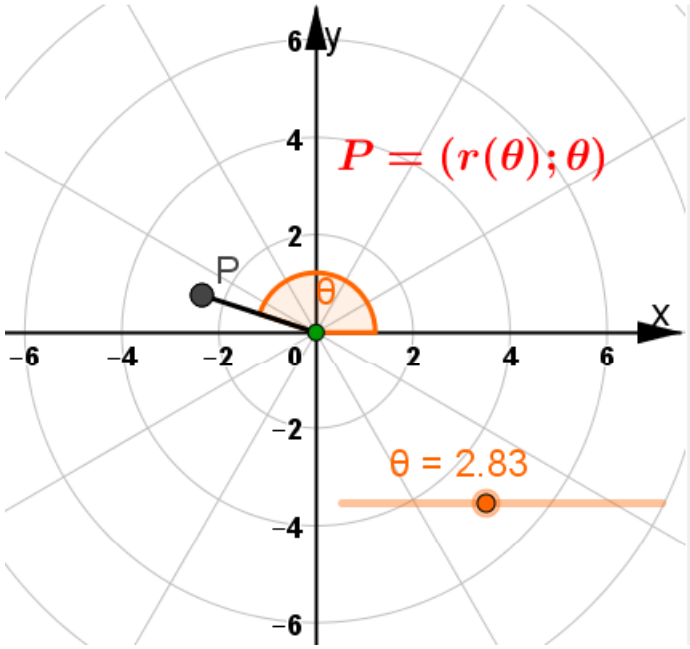
Zur Abszisse θ
gehört die Ordinate $r(\theta)$

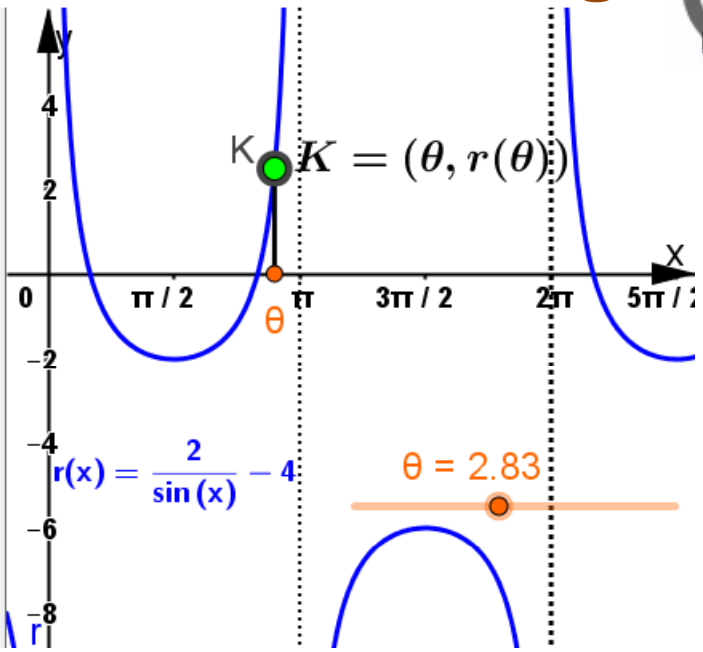


allgemeinere Polarkurven

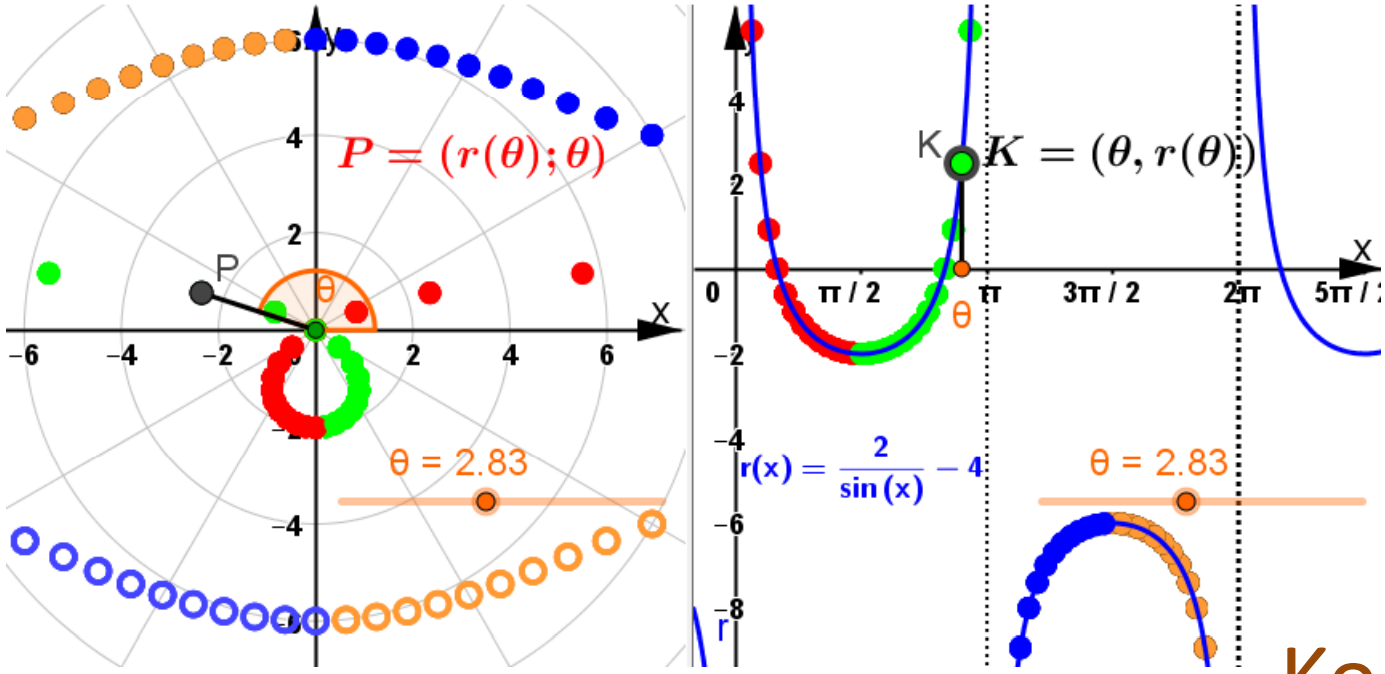


polar-kartesische Darstellung

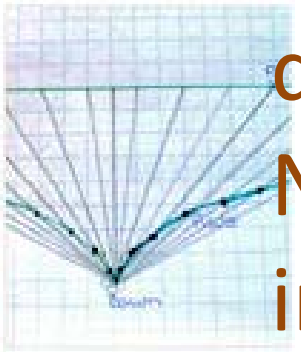




Polarkurven



in SekII und Uni, v.A. Lehramt

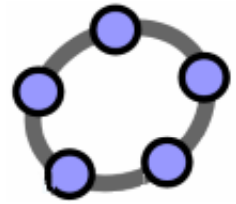


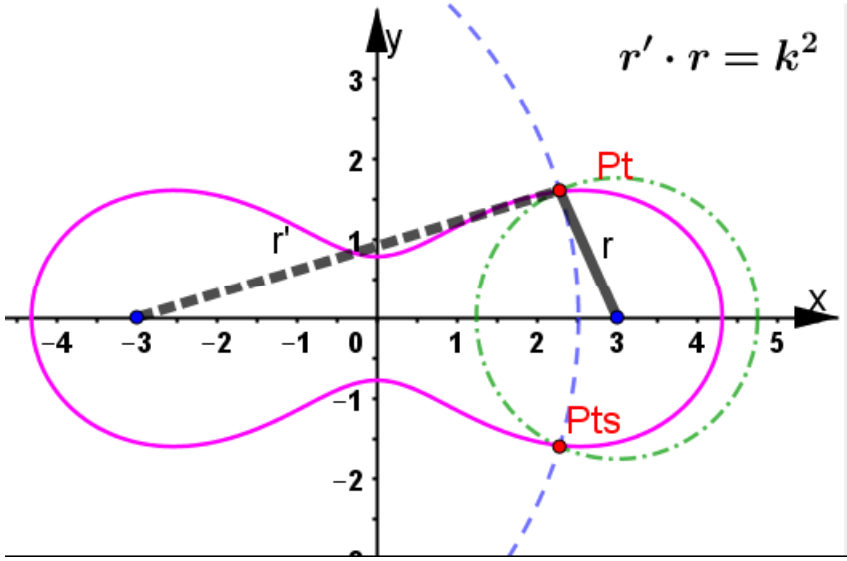
Konchoide des Nikomedes in Klasse 8

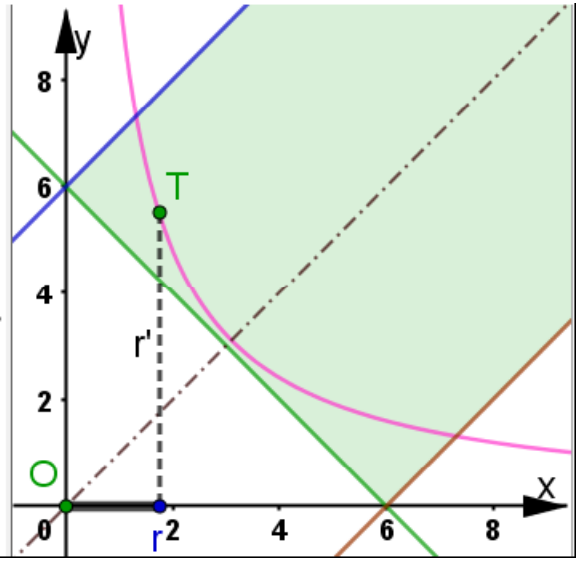
Bipolare Kurven

Die Abstände r' und r von zwei Brennpunkten im Abstand 2e erfüllen eine Gleichung.

$$r' \cdot r = k^2 \rightarrow$$ Cassini‘sche Kurven

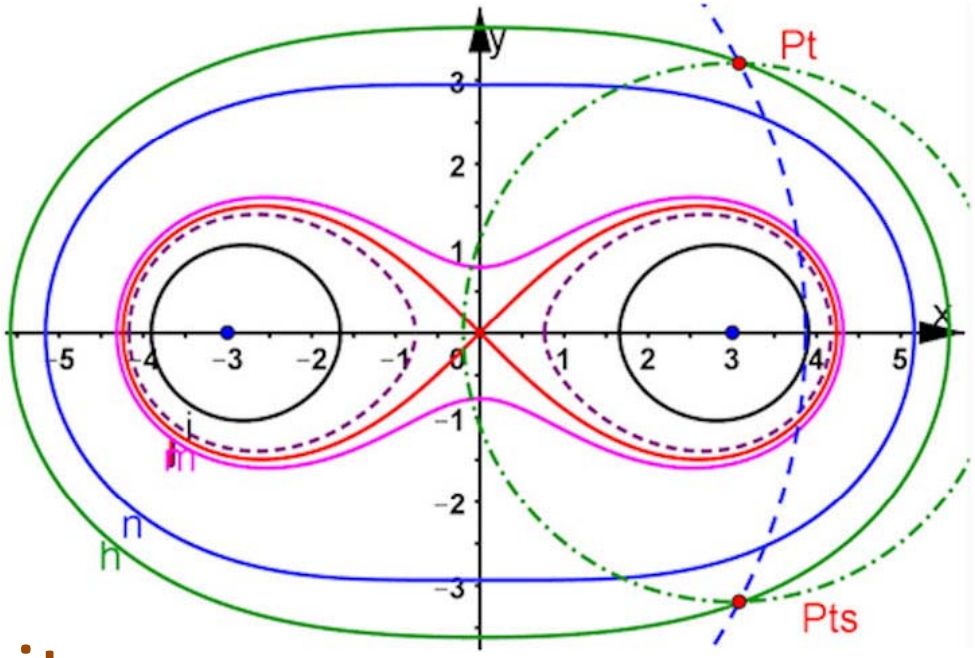


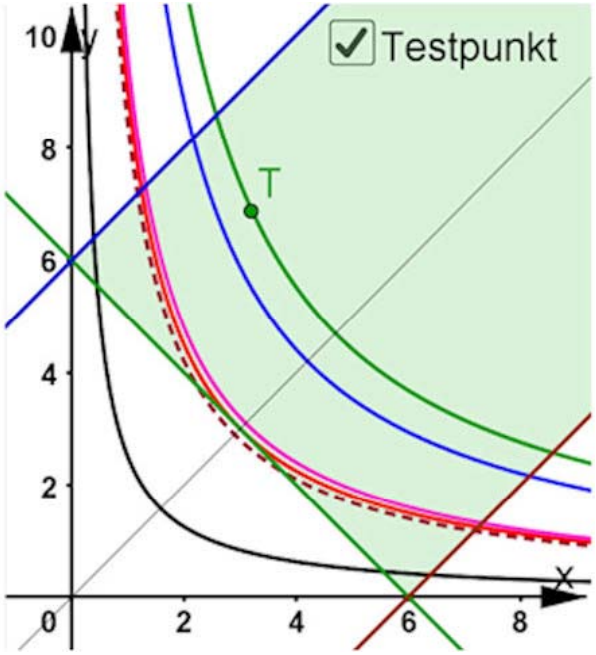




Bipolare Kurven

$r' \cdot r = k^2 \rightarrow$ Cassini‘sche Kurven:





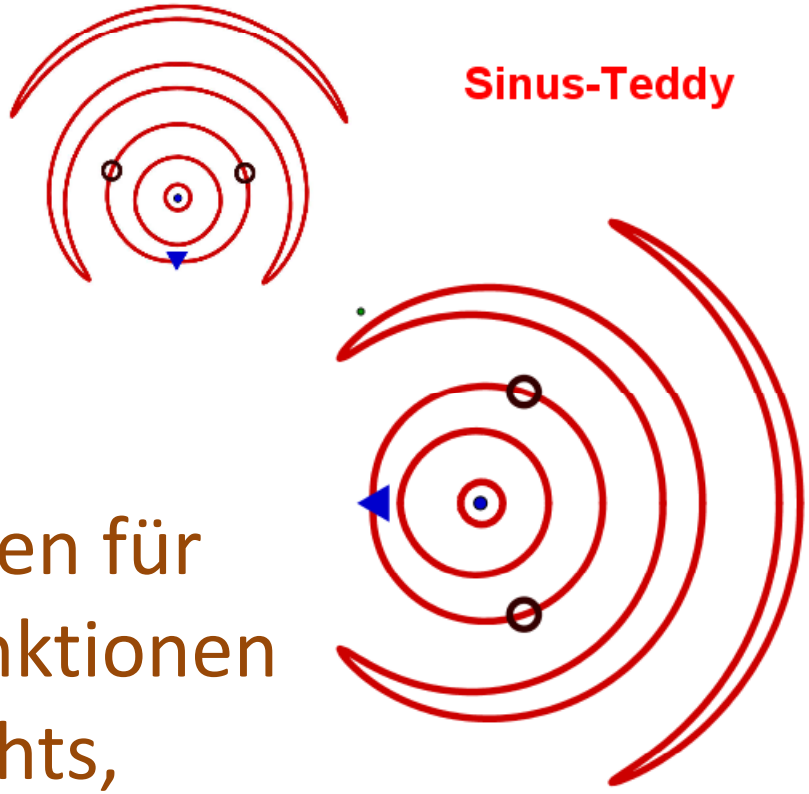
weitere:

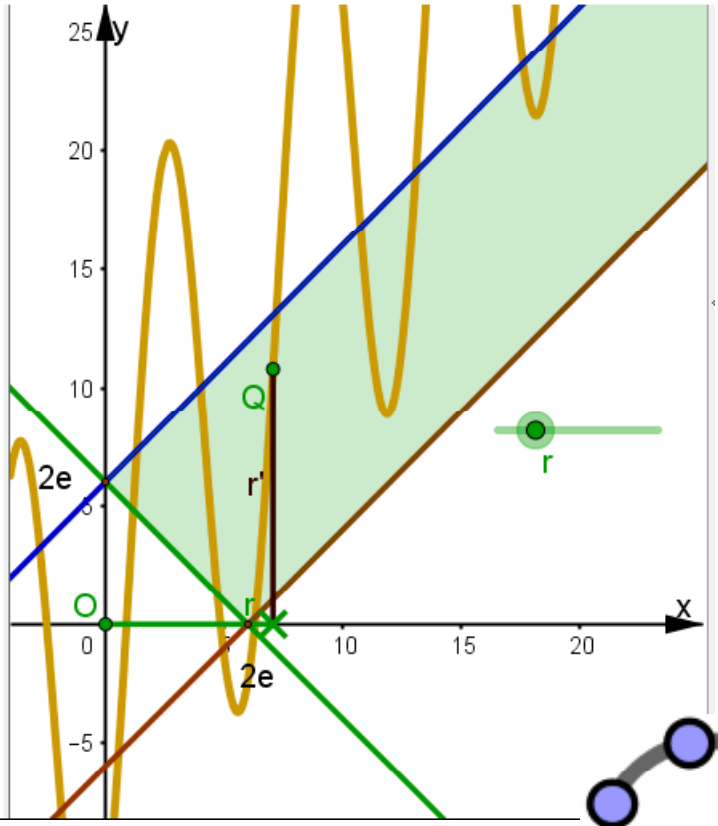
$r' + r = 2a \rightarrow$ Ellipsen

$|r' - r| = 2a \rightarrow$ Hyperbeln

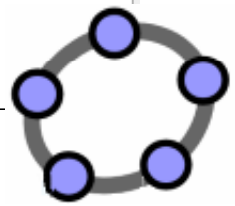
$m \cdot r' + n \cdot r = k \rightarrow$ Descartes‘sche Kurven

eigene bipolare Kurven





Ideen für Funktionen rechts,

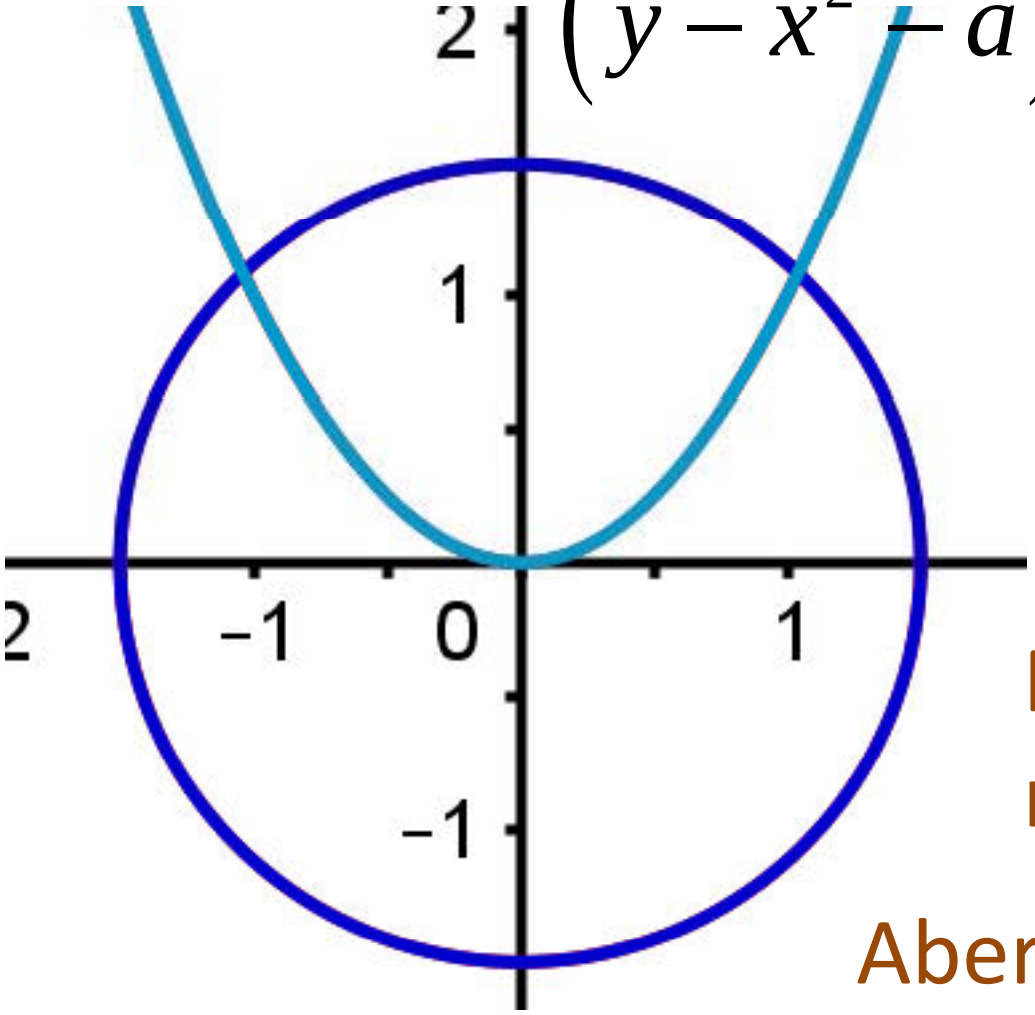


Vorhersagen von Eigenschaften

Deutung von Überraschungen, Kreativität, Stolz……

Raumverwandte Kurven

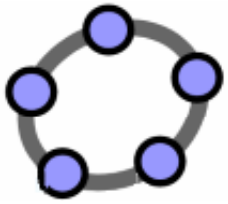
$$\left(y - x^2 - a\right)\left(x^2 + y^2 - r^2\right) = h$$



Produkt aus
Parabel und Kreis
= h

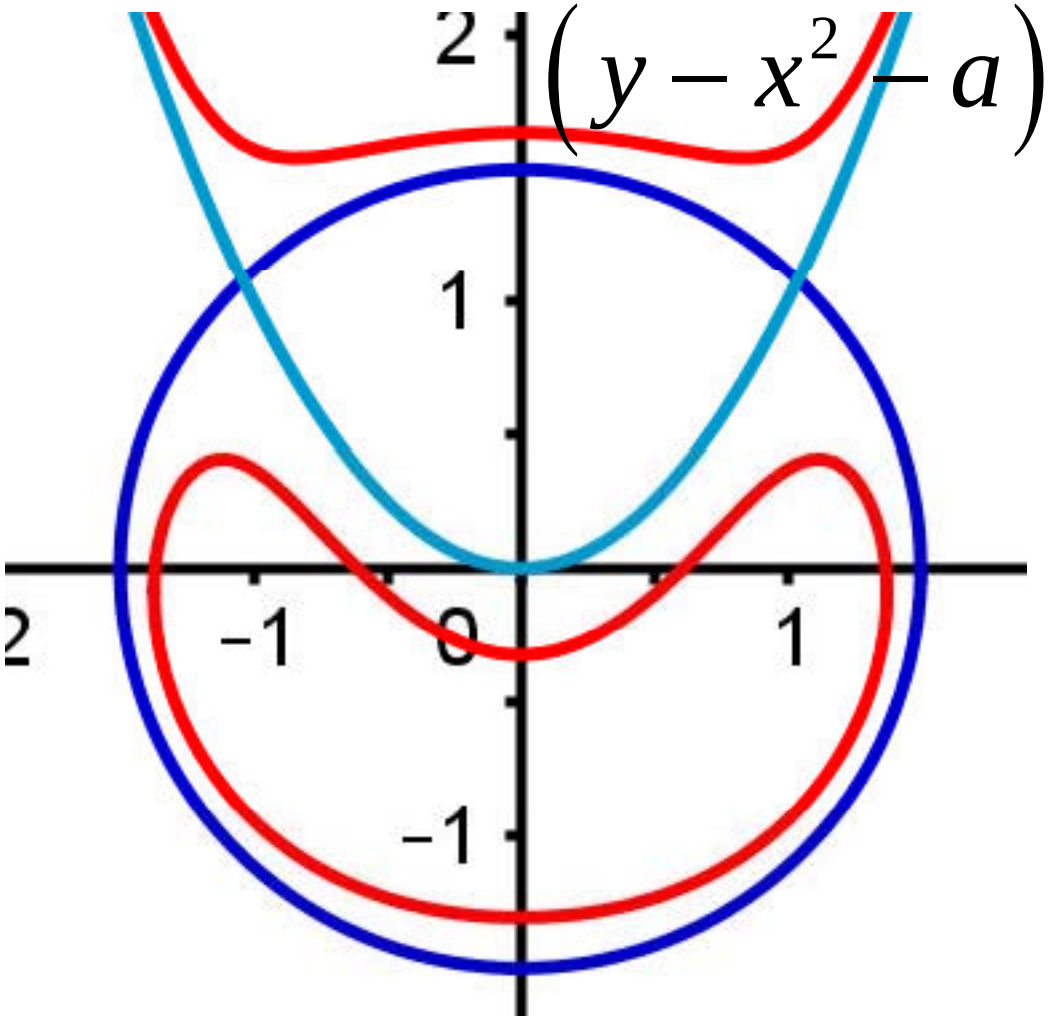
Für h=0 gibt‘s
nichts Neues!

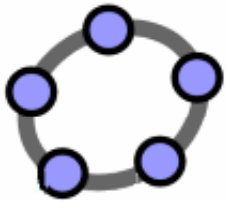
Aber……



Raumverwandte Kurven

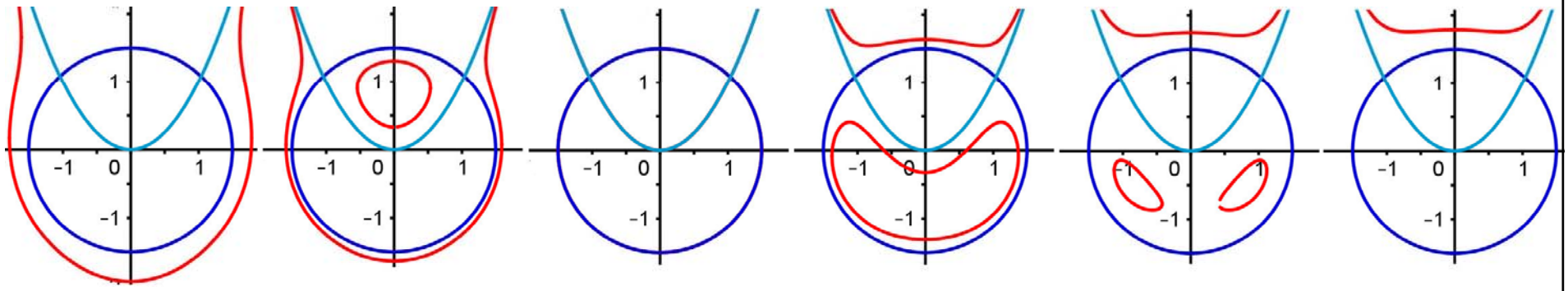
$$\left(y - x^2 - a\right)\left(x^2 + y^2 - r^2\right) = h$$





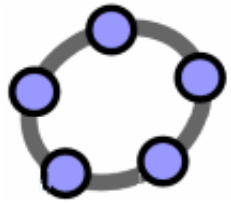
Raumverwandte Kurven

$$\left(y - x^2 - a\right)\left(x^2 + y^2 - r^2\right) = h$$



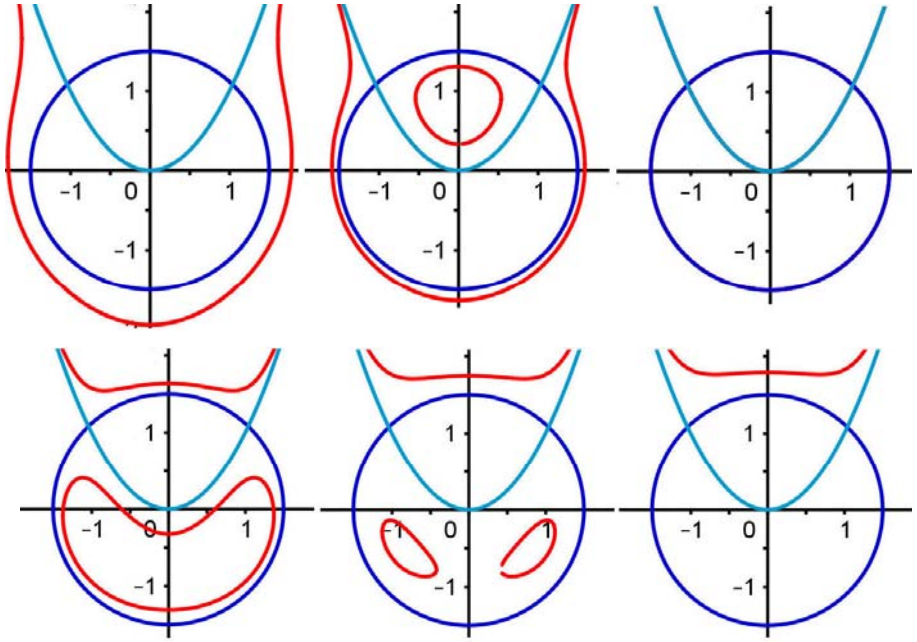
h=-3 h=-0.7 h=0 h=0.7 h=1.4 h=1.7

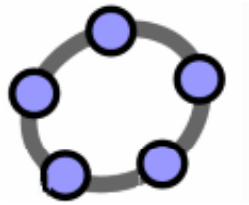
Felix Klein untersuchte Produkte
aus zwei Ellipsen und klassifizierte
die möglichen Formen.

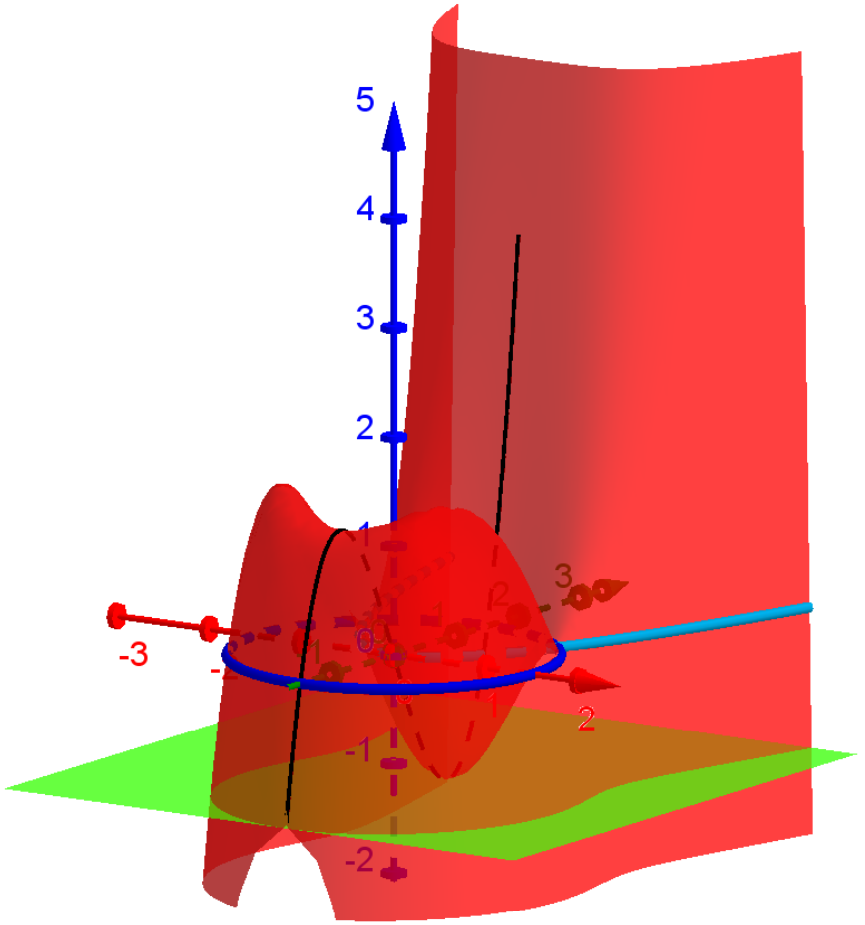


Raumverwandte Kurven

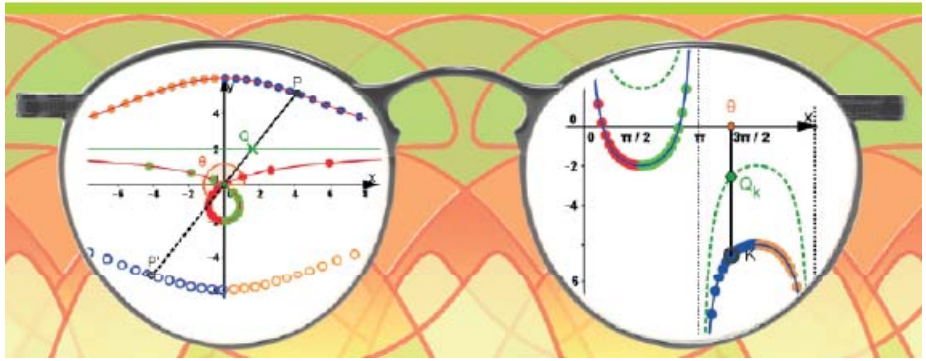
$$d(x,y) = (y - x^2)\ (x^2 + y^2 - 1.5^2)$$

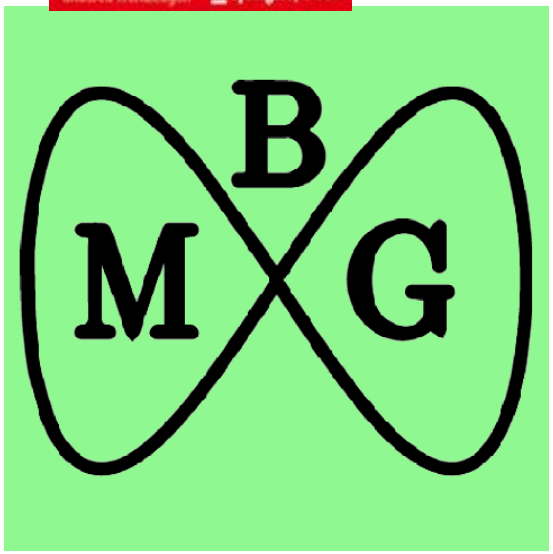




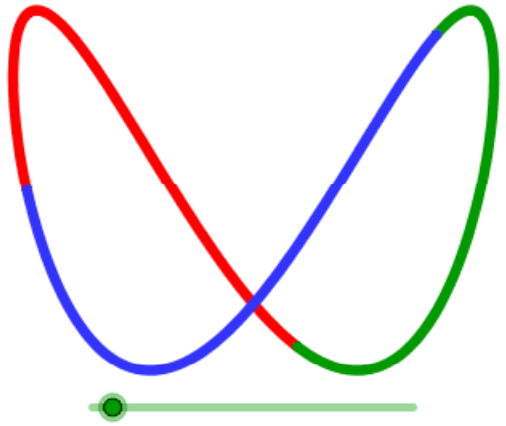


Kurven in **vielen** Perspektiven





Vielen Dank für



Ihre Aufmerksamkeit!